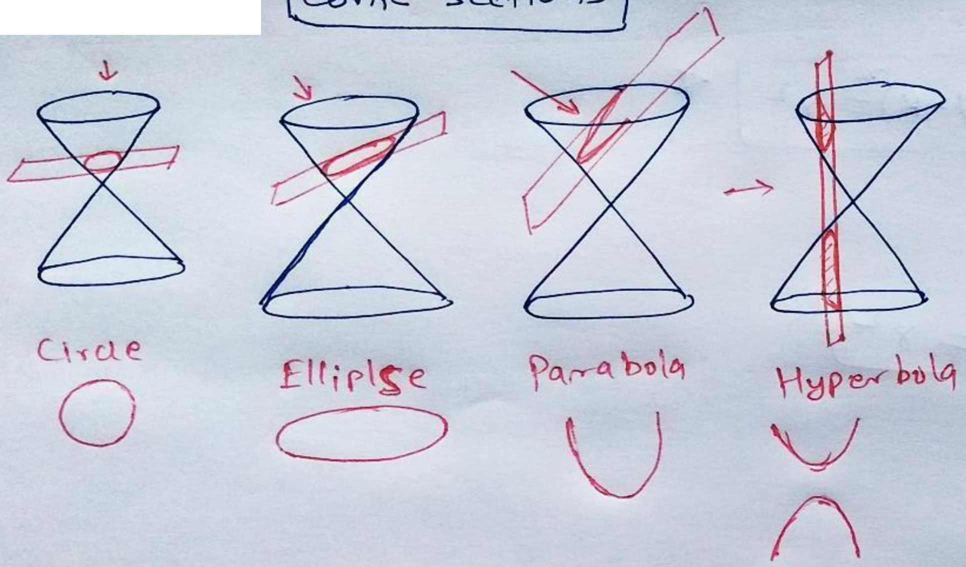
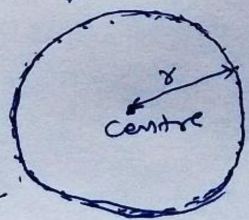
Conic Sections

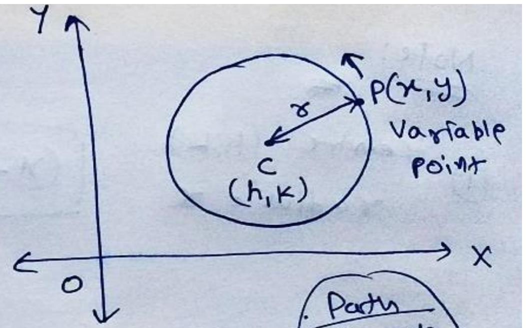


Circle: A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

Fixed Point → centre ✓

Fixed Distance → radius ✓





Centre C(h, K)

radius = r

Path
Graph
Locus
↓
Circle

$\because \; CP = r$

$\Rightarrow \; CP^2 = r^2$

$\Rightarrow \; \boxed{(x-h)^2 + (y-k)^2 = r^2}$

Standard Form of a Circle

e.g. centre (2, 3)
$r = 5$
$(x-2)^2 + (y-3)^2 = 5^2$

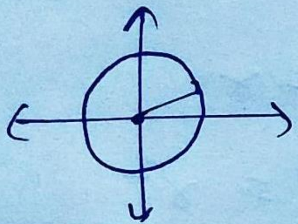
Note:

① Centre (h, k)
radius $= r$

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$ ☆

② Centre $(0, 0)$
radius $= r$

$$\boxed{x^2+y^2=r^2}$$



Exercise 10.1 - Circle

(h, k) r

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Find equation of Circle.

Q.1 Centre $(0, 2)$

$r = 2$

Circle

$(x-0)^2 + (y-2)^2 = 2^2$

$\Rightarrow x^2 + y^2 + \cancel{4} - 4y = \cancel{4}$

$\Rightarrow \boxed{x^2 + y^2 - 4y = 0}$

Q.2 Centre $(-2, 3)$

$r = 4$

Circle.

$(x+2)^2 + (y-3)^2 = 16$

$\Rightarrow x^2 + 4 + 4x$
$+ y^2 + 9 - 6y = 16$

$\Rightarrow \boxed{x^2 + y^2 + 4x - 6y - 3 = 0}$

Q.3 Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$

$r = \frac{1}{12}$

Circle

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$

$\Rightarrow x^2 + \frac{1}{4} - x + y^2 + \frac{1}{16} - \frac{y}{2} = \frac{1}{144}$

$\Rightarrow 144x^2 + 144y^2 - 144x - 72y + 36 + 9 = 0$

$\Rightarrow \boxed{144x^2 + 144y^2 - 144x - 72y + 45 = 0}$

Q.4 $r = \sqrt{2}$, Centre $(1, 1)$

Circle $(x-1)^2 + (y-1)^2 = (\sqrt{2})^2$

$\Rightarrow x^2 - 2x + \cancel{1} + y^2 - 2y + \cancel{1} = \cancel{2}$

$\Rightarrow \boxed{x^2 + y^2 - 2x - 2y = 0}$

Q.5

Centre $(-a, -b)$

$r = \sqrt{a^2 - b^2}$

Circle $(x+a)^2 + (y+b)^2 = \left(\sqrt{a^2-b^2}\right)^2$

$\Rightarrow x^2 + \cancel{a^2} + 2ax + y^2 + b^2 + 2by = \cancel{a^2} - b^2$

$\Rightarrow \boxed{x^2 + y^2 + 2ax + 2by + 2b^2 = 0}$

Conic Sections – Circle

- Centre (h, k)
- $r =$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Complete Square in x = in y

Q.6 $(x+5)^2 + (y-3)^2 = 36 = 6^2$

Centre $= (-5, 3)$

$r = 6$

Q.7 $x^2 + y^2 - 4x - 8y - 45 = 0$

we have to apply "completing the square method"

$x^2 + y^2 - 4x - 8y - 45 = 0$

$\Rightarrow \underline{x^2 - 4x} + \underline{y^2 - 8y} = 45$

$\Rightarrow \underbrace{x^2 - 4x + 4} + \underbrace{y^2 - 8y + 16} = 45 + 4 + 16$

$\Rightarrow (x-2)^2 + (y-4)^2 = 65$

$\Rightarrow (x-2)^2 + (y-4)^2 = (\sqrt{65})^2$

Centre $(2, 4)$ ✓

$r = \sqrt{65}$ ✓

Q.8 $x^2 + y^2 - 8x + 10y - 12 = 0$

$\Rightarrow \underline{x^2 - 8x} + \underline{y^2 + 10y} = 12$

$\Rightarrow \underbrace{x^2 - 8x + 16} + \underbrace{y^2 + 10y + 25} = 12 + 16 + 25$

$\Rightarrow (x-4)^2 + (y+5)^2 = 53 = (\sqrt{53})^2$

$\boxed{\text{Centre } (4, -5)}$ $\boxed{r = \sqrt{53}}$

Q.9 $2x^2+2y^2-x=0$

$\Rightarrow \left(2x^2 - x + 2y^2 = 0\right) \div 2$

$\Rightarrow \underbrace{x^2 - \frac{x}{2} + \frac{1}{16}} + \underbrace{y^2} = 0 + \frac{1}{16}$

$y^2 = (y-0)^2$

$\Rightarrow \left(x-\frac{1}{4}\right)^2 + y^2 = \frac{1}{16} = \left(\frac{1}{4}\right)^2$

Centre $\left(\frac{1}{4}, 0\right)$ ✓

$r = \frac{1}{4}$ ✓

Q.10 – Passing through (4,1) & (6,5)

– Centre on $\boxed{4x+y=16}$

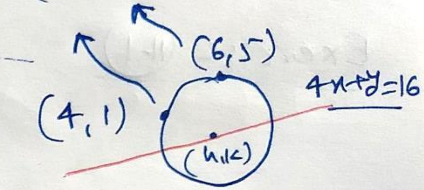
Let the centre be (h,k) ✓

radius $= r$ ✓

$\boxed{4h+k=16}$ — ①

Circle:

$(x-h)^2+(y-k)^2=r^2$



(4,1)

$(4-h)^2+(1-k)^2=r^2$ — ②

(6,5)

$(6-h)^2+(5-k)^2=r^2$ — ③

$\Rightarrow \cancel{h^2}-8h+\cancel{k^2}-2k+17=\cancel{r^2}$

$\Rightarrow \cancel{h^2}-12h+\cancel{k^2}-10k+61=\cancel{r^2}$

$\ \ -\qquad +\qquad -\qquad +\qquad -$

$4h+8k-44=0$

$\Rightarrow \boxed{h+2k=11}$ — ④

$h=11-2k$

By eqⁿ ① & ④:

$\Rightarrow 4(11-2k)+k=16$

$\Rightarrow 4(11-2K)+K=16$

$\Rightarrow 44-8K+K=16$

$\Rightarrow 28=7K$

$\Rightarrow \boxed{K=4}$

$h=11-2K$

$h=11-2\times4=3$

Centre $(h,K)\equiv(3,4)$

By eqⁿ ②: $(4-h)^2+(1-K)^2=r^2$

$\Rightarrow (4-3)^2+(1-4)^2=r^2$

$\Rightarrow 1+9=r^2$

$\Rightarrow r^2=10$

$r=\sqrt{10}$

Circle: $(x-3)^2+(y-4)^2=10$

Q.11 Passing through $(2,3)$, $(-1,1)$

Centre on $x-3y-11=0$

Let centre (h,K) → $h-3K-11=0$ — ①

radius $=r$

Circle: $\boxed{(x-h)^2+(y-K)^2=r^2}$

$(2,3)\rightarrow (2-h)^2+(3-K)^2=r^2$ — ②

$(-1,1)\rightarrow (-1-h)^2+(1-K)^2=r^2$ — ③

$11=6h+4K$ — ④

By eqⁿ ① & ④: $(h,K)\equiv\left(\frac{7}{2},\frac{-5}{2}\right)$

By eqⁿ ② $r=\sqrt{\frac{65}{2}}$

Circle: $\boxed{\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{65}{2}}$

) Circle.

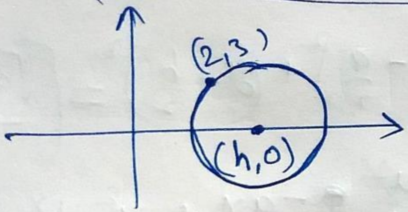
Centre (h, k) radius $= r$

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

Q.12 $r=5$ ✓

Centre on x-axis → Centre $(h, 0)$

Passing through $(2, 3)$



~~Let~~ Circle $(x-h)^2+y^2=r^2=5^2$

$\Rightarrow \boxed{x^2-2hx+h^2+y^2=25}$

$(2,3)$ satisfy

$\Rightarrow 4-4h+h^2+9=25$

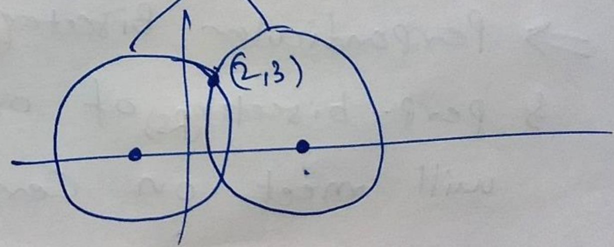
$\Rightarrow h^2-4h-12=0$

$\Rightarrow h^2-6h+2h-12=0$

$\Rightarrow h(h-6)+2(h-6)=0$

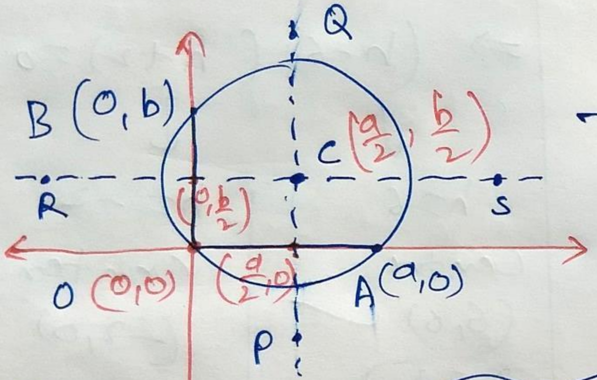
$\Rightarrow (h-6)(h+2)=0$

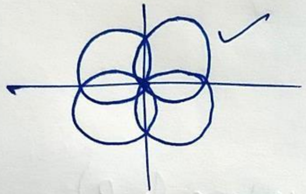
$h=6$	$h=-2$
Centre $(6, 0)$ ✓	Center $(-2, 0)$
$r=5$	$r=5$
$\boxed{(x-6)^2+(y-0)^2=25}$	$\boxed{(x+2)^2+y^2=25}$

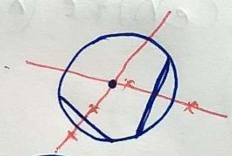


Q.13 Circle passing through (0,0)
making intercepts a & b on axes.

x - intercept   y - intercept







Class - 9 → Perpendicular Bisector of any chord of a circle always passes through the 'centre'.

⇒ Perpendicular Bisector (PQ) of OA (chord) & perp. bisector (RS) of OB (chord) will meet on Centre 'C' $\left(\frac{a}{2}, \frac{b}{2}\right)$

Centre C $\left(\frac{a}{2}, \frac{b}{2}\right)$

O(0,0)

radius = r = OC

$r = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-0\right)^2}$

$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$

$\boxed{r = \frac{\sqrt{a^2+b^2}}{2}}$

Circle

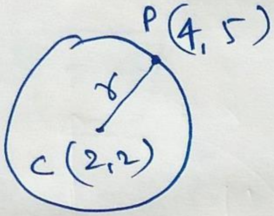
$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left[\frac{\sqrt{a^2+b^2}}{2}\right]^2$

$\Rightarrow x^2 + \cancel{\frac{a^2}{4}} - ax + y^2 + \cancel{\frac{b^2}{4}} - by = \cancel{\frac{a^2+b^2}{4}}$

$\Rightarrow \boxed{x^2 + y^2 - ax - by = 0}$

Q.14 Centre $(2,2)$

Passing through $(4,5)$



$$r = CP = \sqrt{(4-2)^2 + (5-2)^2}$$

$$r = \sqrt{4+9}$$

$$r = \sqrt{13}, \quad \text{centre } (2,2)$$

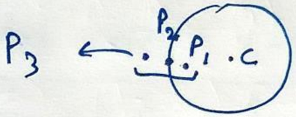
Circle:
$$(x-2)^2 + (y-2)^2 = (\sqrt{13})^2$$

$$\Rightarrow x^2 - 4x + 4 + y^2 - 4y + 4 = 13$$

$$\Rightarrow \boxed{x^2 + y^2 - 4x - 4y - 5 = 0}$$

Q.15 Circle $\boxed{x^2 + y^2 = 25}$

Position of point $P(-2.5, 3.5) = ?$



radius $= r$

outside $P_3C > r$

on the circle $P_2C = r$

inside. $P_1C < r$

$$(x-0)^2 + (y-0)^2 = 5^2$$

☆ centre $(0,0)$ c

☆ $r = 5$

Distance b/w point P & centre c

$P \to (-2.5, 3.5)$, $c \to (0,0)$

$$CP = \sqrt{(-2.5)^2 + (3.5)^2}$$

$$CP = \sqrt{6.25 + 12.25}$$

$$CP = \sqrt{18.50}$$

$CP = \sqrt{18.50}$

$r = 5$

$\Rightarrow \boxed{CP < r}$

$16 < 18.5 < 25$

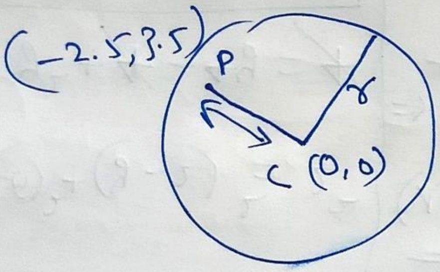
$\downarrow \quad \downarrow \quad \downarrow$

$4 \longleftrightarrow 5$

$CP^2 < r^2$

$\downarrow \quad \downarrow$

$18.5 < 25$



P(-2.5, 3.5)

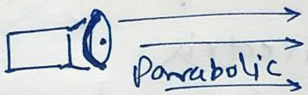
$\boxed{\text{inside the circle}}$

PARABOLA for Exercise 10.2

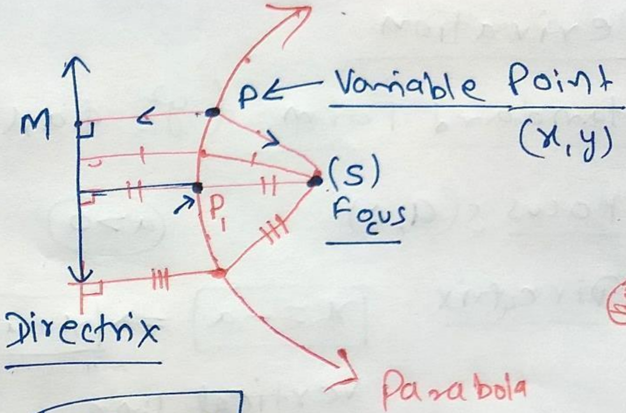
Para bola

↓ ↓

for throwing →

Torch → 

Dish Antenna 



$$\boxed{SP = PM}$$

← Parabola for

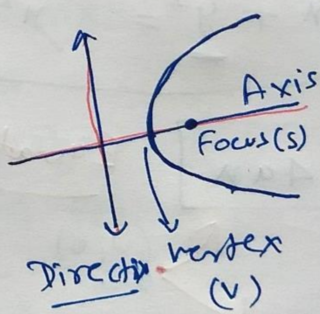
$$\boxed{e = \frac{SP}{PM} = 1}$$

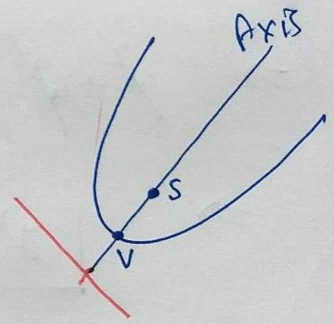
↑ eccentricity

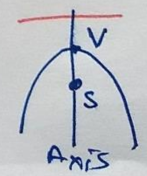
Definition: set of all points on a plane that are equidistant from a fixed point (Focus) & from a fixed line (Directrix).

Axis

Vertex







Derivation

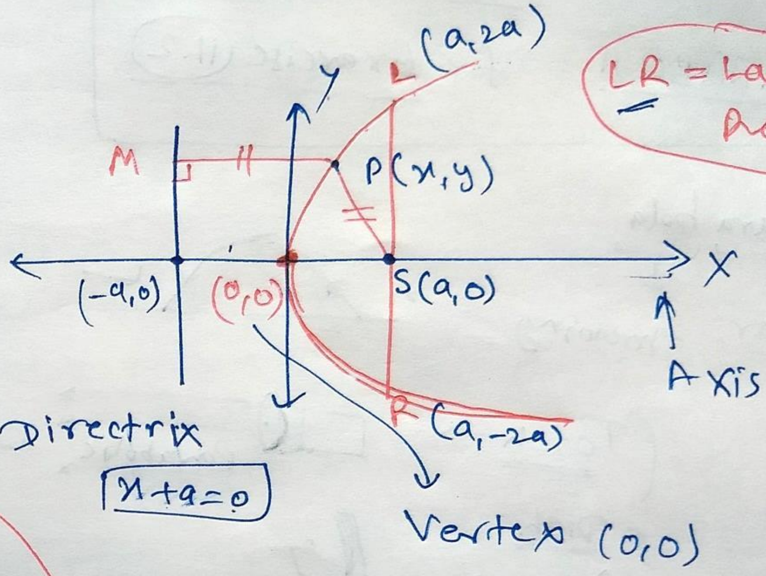
Standard Form ($y^2 = 4ax$)

LR = Latus Rectum

Focus $S(a,0)$ $\quad a>0$

Directrix $\boxed{x=-a} \Rightarrow \boxed{x+a=0}$

vertical line.



$$\boxed{SP = PM}$$

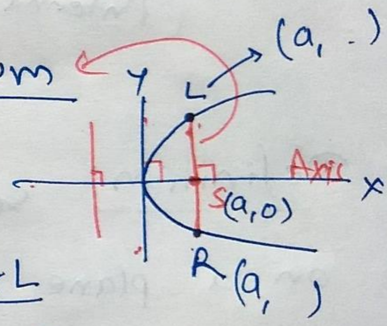
$$\Rightarrow \left(\sqrt{(x-a)^2+(y-0)^2}\right)^2 = \left(\left|\frac{x+a}{\sqrt{1^2+0^2}}\right|\right)^2$$

$$\Rightarrow \cancel{x^2}+\cancel{a^2}-2ax+y^2 = \cancel{x^2}+\cancel{a^2}+2ax$$

$$\Rightarrow \boxed{y^2 = 4ax}$$ Equation of parabola

(0,0)

Latus Rectum



$y^2 = 4ax$

(a)

For L

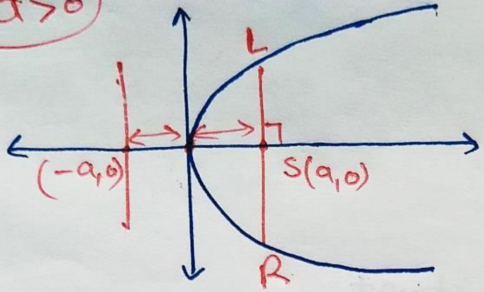
$y^2 = 4a.a$

$y = \pm 2a \rightarrow y = +2a$

$\rightarrow y = -2a$

$\boxed{\text{Length of latus Rectum} = 4a}$

I $\boxed{y^2 = 4ax}$ $(a>0)$



Position ⇒ Rightward

Vertex ⇒ $(0,0)$

Focus ⇒ $(a,0)$

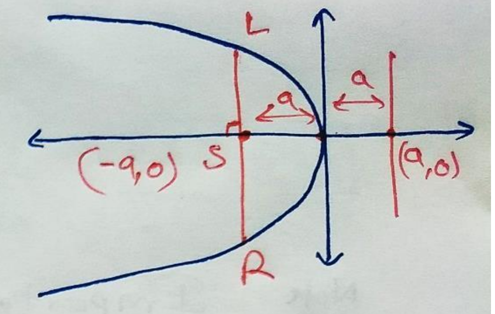
Directrix ⇒ $\boxed{x=-a}$

Axis ⇒ x-axis

Symmetric About ⇒ x-axis

Latus Rectum (LR)
- → Equation ⇒ $\boxed{x=a}$
- → Length ⇒ $4a$

II $\boxed{y^2 = -4ax}$



Position ⇒ Leftward

Vertex ⇒ $(0,0)$

Focus ⇒ $S(-a,0)$

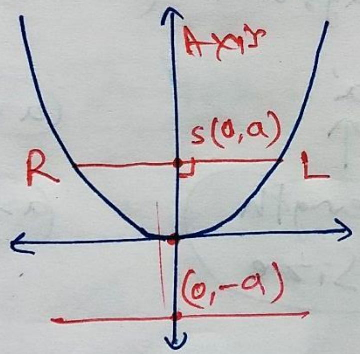
Directrix ⇒ $\boxed{x=a}$

Axis ⇒ x-axis

Symmetric About ⇒ x-axis

Latus Rectum
- → Equation ⇒ $\boxed{x=-a}$
- → length ⇒ $4a$

III $\boxed{x^2 = 4ay}$



Position ⇒ Upward

Vertex ⇒ $(0,0)$

Focus ⇒ $S(0,a)$

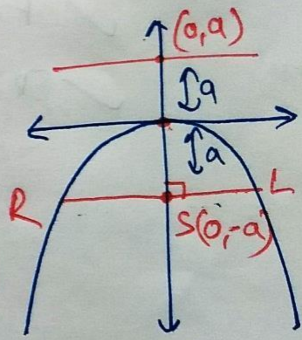
Directrix ⇒ $\boxed{y=-a}$

Axis ⇒ y-axis

Sym. about ⇒ y-axis

Latus Rectum
- → Equation ⇒ $\boxed{y=a}$
- → length ⇒ $4a$

IV $\boxed{x^2 = -4ay}$



Position ⇒ Downward

Vertex ⇒ $(0,0)$

Focus ⇒ $S(0,-a)$

Directrix ⇒ $\boxed{y=a}$

Axis ⇒ y-axis

Sym. about ⇒ y-axis

Latus Rectum (LR)
- → Equation ⇒ $\boxed{y=-a}$
- → length ⇒ $4a$

Note

Important things for Derivation

↓

✓ Focus

✓ Directrix

Important things for Question Solving

↓

☆ (1) Diagram (Exact)

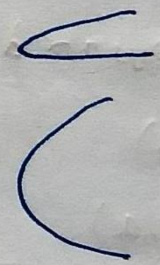
(2) 'a'

↑

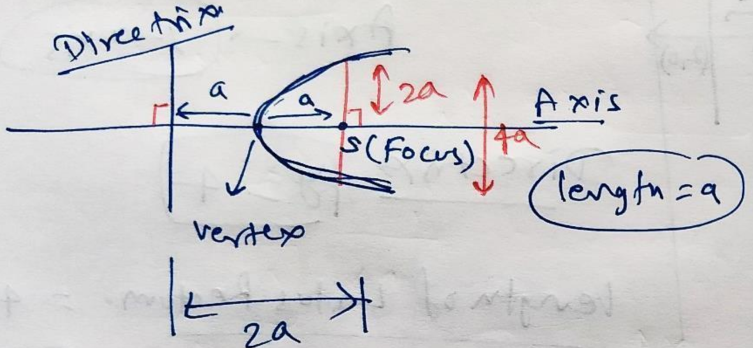
length / Size

$a <<$

$a >>$



PARABOLA

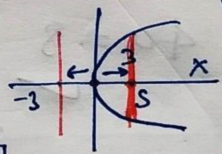


length = a

Q.1 $\boxed{y^2 = 12x} \Rightarrow \boxed{y^2 = 4ax}$

$4a = 12$

$a = 3$



Coordinate of focus $\Rightarrow S(3,0)$

Axis of Parabola $\Rightarrow$ x-axis $\boxed{y=0}$

Eqn. of Directrix $\Rightarrow \boxed{x=-3}$

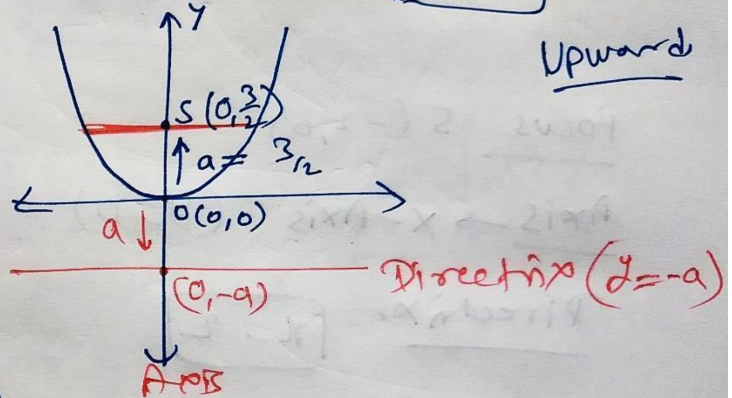
length of latus Rectum $\Rightarrow 4a = 4 \times 3$

$= 12$

Q.2 $\boxed{x^2 = 6y} \rightarrow \boxed{x^2 = 4ay}$

$6 = 4a \Rightarrow \boxed{a = \frac{3}{2}}$

Upward



Focus $= S\left(0, \frac{3}{2}\right)$

Axis $\rightarrow$ y-axis $(x=0)$

Directrix $\rightarrow \boxed{y=-a}$

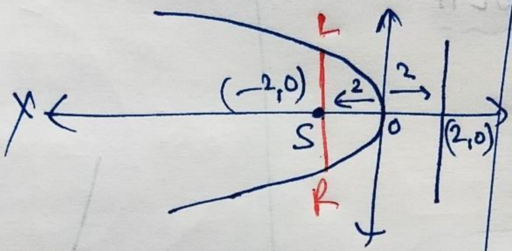
length of Latus Rectum

$\rightarrow 4a = 6$

Q.3 $\boxed{y^2 = -8x} \longleftrightarrow \boxed{y^2 = -4ax}$

$+8 = +4a$

$\boxed{a = 2}$



Focus $S(-2,0)$

Axis → X-Axis $(y=0)$

Directrix $\boxed{x = 2}$

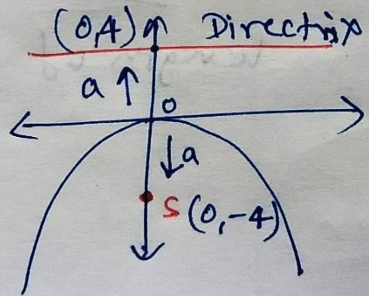
Length of Latus Rectum $(LR) = 4a = 8$

Q.4 $\boxed{x^2 = -16y} \longleftrightarrow \boxed{x^2 = -4ay}$

$-16 = -4a$

$4a = 16$

$\boxed{a = 4}$



Continue Q.4

Answer focus $S(0,-4)$

Axis → $\boxed{y\text{-axis}}$

Directrix $\boxed{y = 4}$

Length of Latus Rectum $= 4a$
$= 16$

Q.

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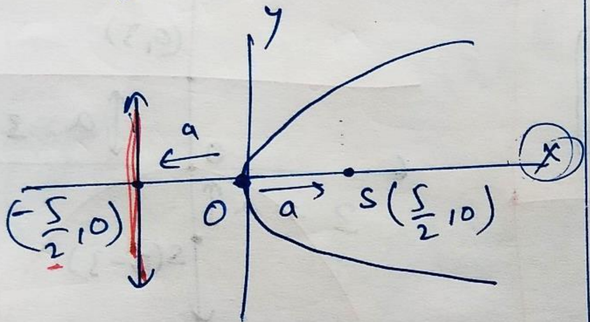
Q.5

$\boxed{y^2 = 10x} \longleftrightarrow \boxed{y^2 = 4ax}$

$10 = 4a$

$\Rightarrow a = \frac{5}{2}$



Focus $= S(\frac{5}{2}, 0)$

Axis : x-axis $(y = 0)$

Directrix : $\boxed{x = -\frac{5}{2}}$

Length of Latus Rectum $= 4a$

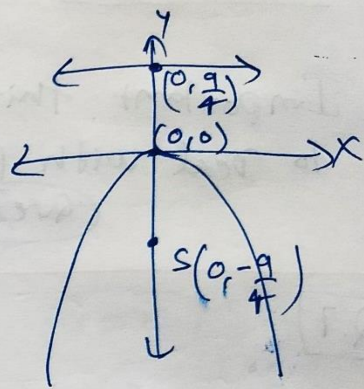
$= 10$

Q.6

$x^2 = -9y \longleftrightarrow \boxed{x^2 = -4ay}$

$9 = 4a$

$\boxed{a = \frac{9}{4}}$



Focus $= S(0, -\frac{9}{4})$

Axis $\rightarrow$ y-axis $(x = 0)$

Directrix $\rightarrow$ $y = \frac{9}{4}$

Length of Latus Rectum $= 4a$

$= 9$ units.

Parabola

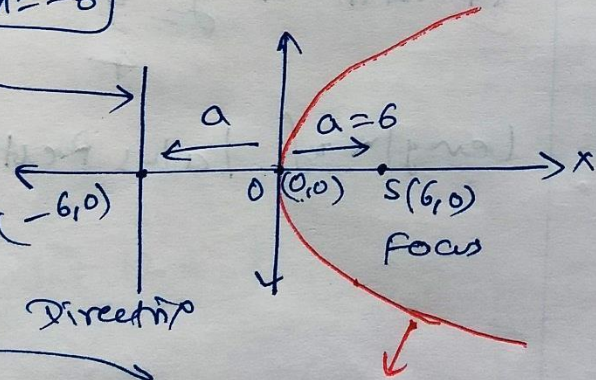
☆ Important things to Deal with parabola Question → ① Diagram (Exact) ② a

$y^2 = 4ax$	$x^2 = 4ay$
$y^2 = -4ax$	$x^2 = -4ay$

Q.7

Focus $S(6,0)$ → on ⊕ x-axis

Directrix $\boxed{x = -6}$ → vertical



$a = 6$

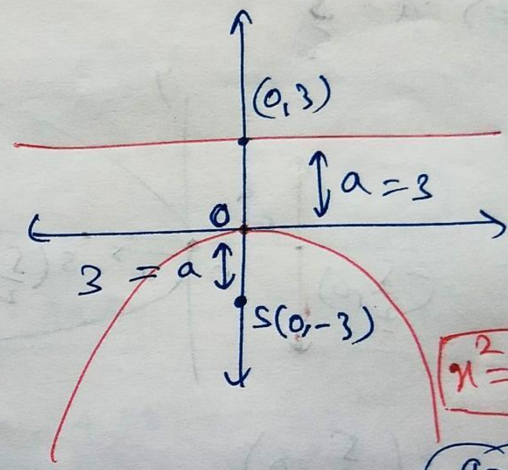
Parabola $\boxed{y^2 = 4ax}$

$y^2 = 4 \times 6 . x$

$\boxed{y^2 = 24x}$

⑧ Focus $(0,-3)$

Directrix $\boxed{y = 3}$ → Horizontal



$\boxed{x^2 = -4ay}$

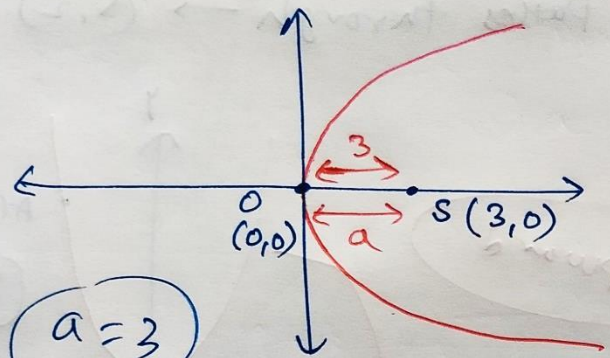
$a = 3$

Parabola → $x^2 = -4(3).y$

$\boxed{x^2 = -12y}$

Q.9 vertex $(0,0)$

Focus $S(3,0)$



$a = 3$

Rightward

$y^2 = 4ax$

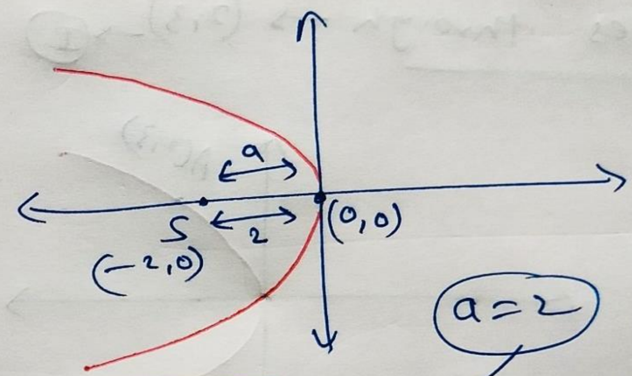
$\Rightarrow a = 3$

Parabola, $y^2 = 4(3)(x)$

$\boxed{y^2 = 12x}$

Q.10 Vertex $(0,0)$

Focus $(-2,0)$



$a = 2$

$\boxed{y^2 = -4ax}$

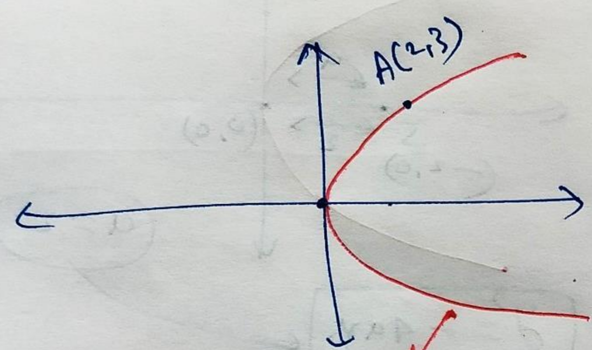
$y^2 = -4(2)x$

$\boxed{y^2 = -8x}$

Q.11 Vertex $(0,0)$

Axis $\rightarrow$ x-axis

Passes through $\rightarrow (2,3) \rightarrow$ I



$$\boxed{y^2 = 4ax}$$

$(2,3)$ will satisfy

$\Rightarrow (3)^2 = 4a.(2)$

$\Rightarrow 9 = 8.a$

$\Rightarrow \boxed{a = \frac{9}{8}}$

Parabola

$y^2 = 4ax$

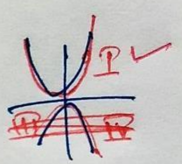
$\Rightarrow y^2 = 4\left(\frac{9}{8}\right)x$

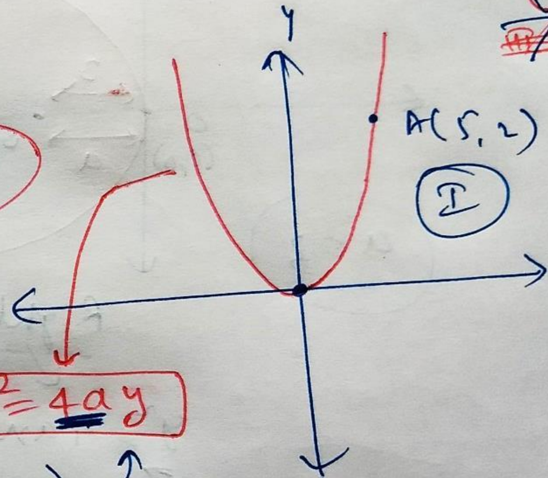
$\Rightarrow \boxed{y^2 = \frac{9}{2}x}$

Q.12 Vertex $(0,0)$ Axis

Symmetric about $\rightarrow$ y-axis

Passes through $\rightarrow (5,2)$





upward

$$\boxed{x^2 = 4ay}$$

$A(5,2)$

$\Rightarrow (5)^2 = 4a(2)$

$\Rightarrow \boxed{\frac{25}{2} = 4a}$

Parabola,

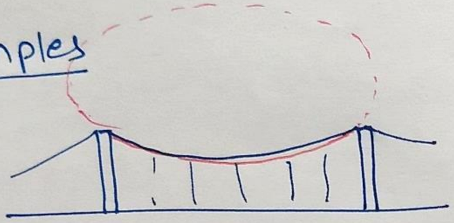
$\boxed{x^2 = \frac{25}{2}y}$

$\boxed{2x^2 = 25y}$

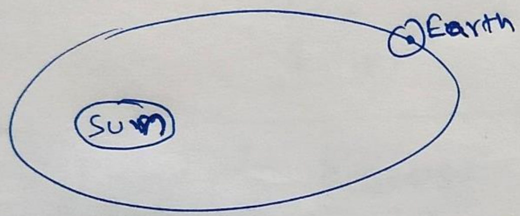
ELLIPSE (before Exercise-10.3

Practical Examples

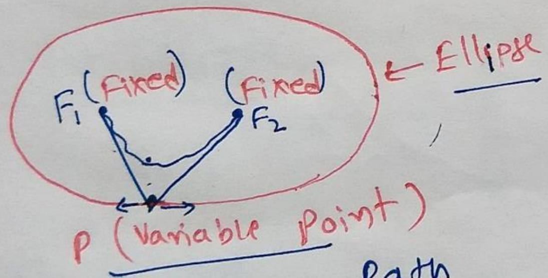
①



②



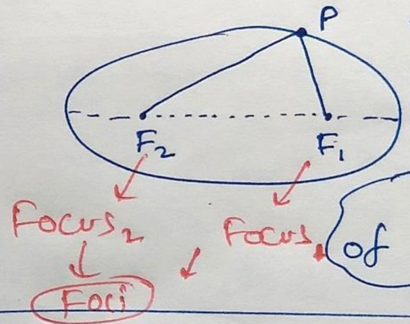
③



Path
(Graph)
Locus

Definition - I.

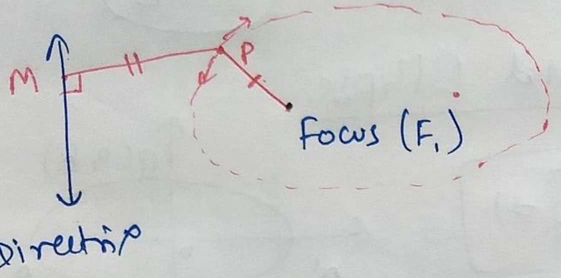
Ellipse is the locus of a moving point (P) whose sum of distances ($PF_1 + PF_2$) from two fixed points (F_1 & F_2) is always constant.



$$\boxed{\begin{aligned} PF_1 + PF_2 &= \text{Constant} \\ &= 2a > F_1F_2 \end{aligned}}$$

For the derivation of standard forms

Definition - II Ellipse is the locus of a moving point (P) such that ratio of distance from P to a fixed point (F_1) & distance from P to a fixed line (Directrix) is constant (e) and always less than 1.



$$\boxed{e = \frac{PF_1}{PM}} < 1$$

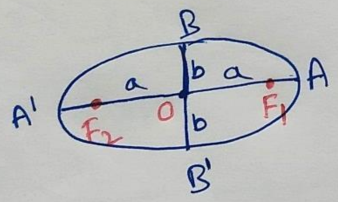
eccentricity

Constant

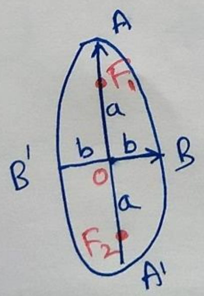
For Ellipse

Terms Related to Ellipse

Horizontal Ellipse



Vertical Ellipse



Major Axis $= AA' = 2a$

$\boxed{a > b}$

Minor Axis $= BB' = 2b$

Semi Major Axis $= OA = OA' = a$

Semi minor Axis $= OB = OB' = b$

Centre (O) ← point of intersection of major & minor Axes.

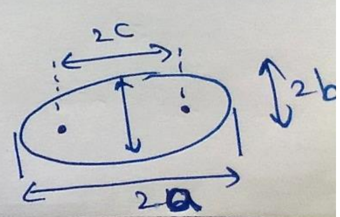
Vertices → A & A' only

↑

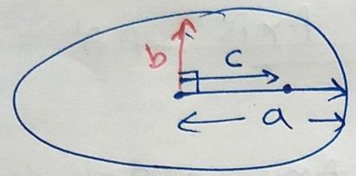
[end points of major Axis (AA')]

Distance b/w Foci $= (F_1F_2) = 2c$

$\boxed{OF_1 = OF_2 = c}$

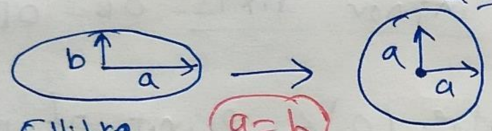


After Derivation



$$\boxed{c = \sqrt{a^2 - b^2}} \star$$

Special case.



a = b

Ellipse $\quad a = b \quad$ Circle.

$c = \sqrt{a^2 - b^2}$ $\qquad$ $c = 0$

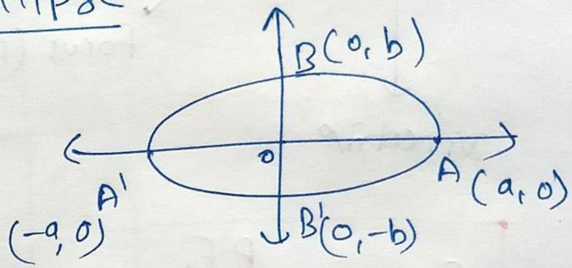
Eccentricity $= \boxed{e = \dfrac{c}{a}} < 1$

$a > b, c$ $\qquad$ $e < 1$

Standard Forms. $\quad a > b$

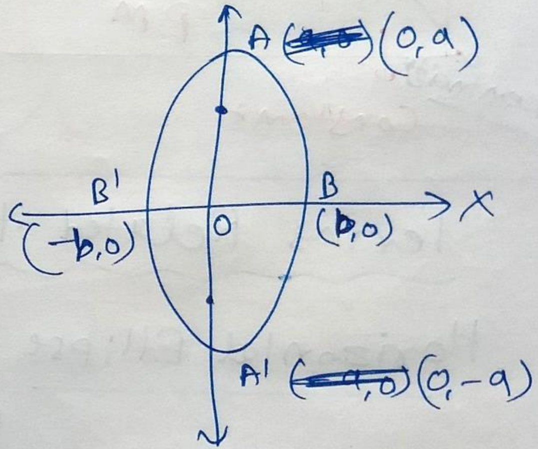
Horizontal Ellipse

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$



Vertical Ellipse

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

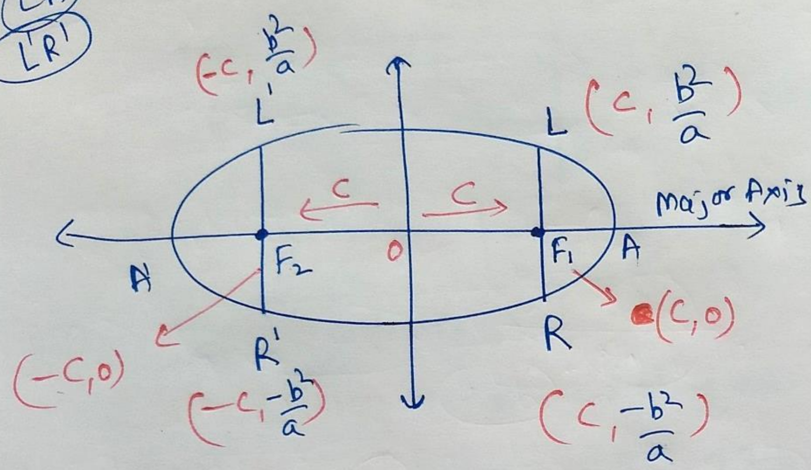


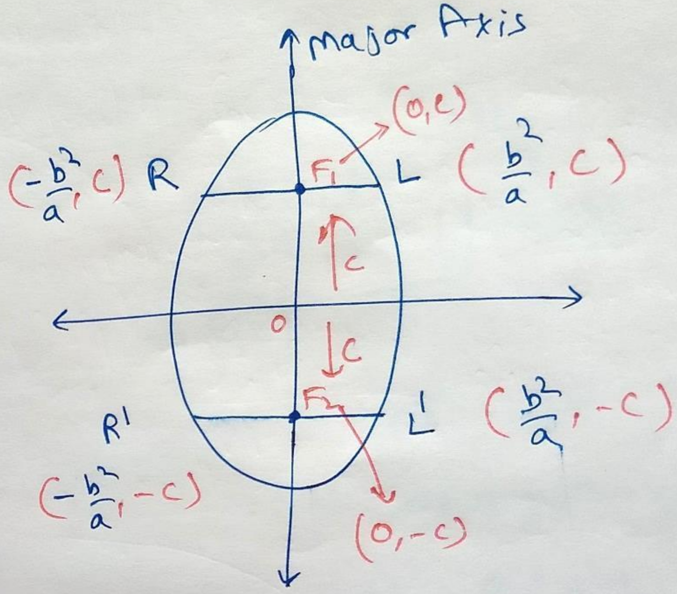
Latus Rectum $\perp$ Major Axis

(Passes through Focus)

LR

L'R'





$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$a > b$

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

$a > b$

for 'L' $\boxed{x = c}$

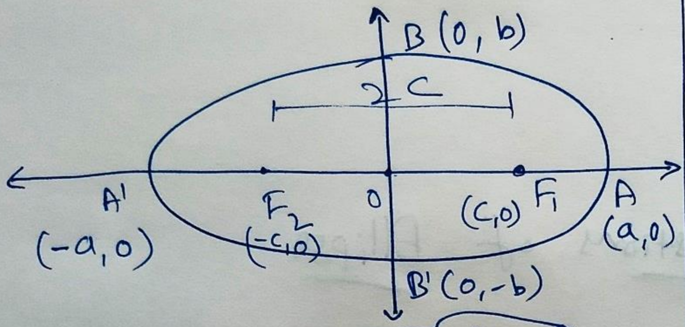
Solve $\boxed{y = \frac{b^2}{a}}$

☆ $\boxed{\text{length of Latus Rectum} = \frac{2b^2}{a}}$

$L(c, \frac{b^2}{a})$

$R(c, -\frac{b^2}{a})$

Horizontal Ellipse ($a > b$)



EQUATION ⇒ $\boxed{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$

centre ⇒ $(0, 0)$

Major Axis ⇒ x-axis → $AA' \to 2a$

Minor Axis ⇒ y-axis → $BB' \to 2b$

Vertices ⇒ $A(a, 0)$, $A'(-a, 0)$

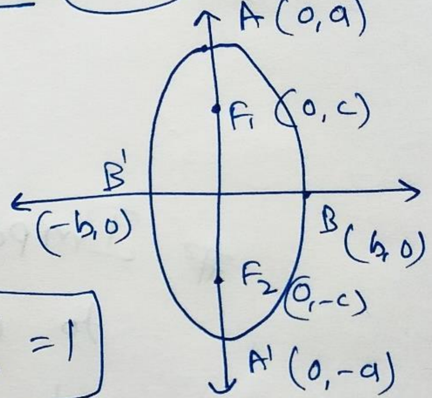
Foci ⇒ $F_1(c, 0)$, $F_2(-c, 0)$

Length of Latus Rectum $= \dfrac{2b^2}{a}$

Eccentricity $= e = \dfrac{c}{a}$

$$c = \sqrt{a^2 - b^2}$$

Vertical Ellipse ($a > b$)



EQUATION $\boxed{\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1}$

Centre ⇒ $(0, 0)$

Major Axis → Y-axis → $AA' = 2a$

Minor Axis → X-axis → $BB' = 2b$

Vertices → $A(0, a)$ $A'(0, -a)$

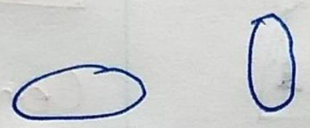
Foci → $F_1(0, c)$, $F_2(0, -c)$

Length of Latus Rectum $= \dfrac{2b^2}{a}$

Eccentricity $= e = \dfrac{c}{a}$

$$c = \sqrt{a^2 - b^2}$$

Important things to deal with Questions of Ellipse

- Diagram 
- a, b, c
  - any two (a,b, b,c, a,c)

Ellipse

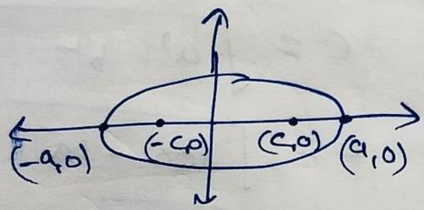
Q.1 $\boxed{\frac{x^2}{36} + \frac{y^2}{16} = 1}$

larger

Horizontal Ellipse

$a > b$

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$



$a = 6, \; b = 4$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{36 - 16} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

① Foci $(\pm c, 0) = (\pm 2\sqrt{5}, 0)$

② Vertices $(\pm a, 0) = (\pm 6, 0)$

③ length of Major Axis $= 2a = 12$

④ length of Minor Axis $= 2b = 8$

⑤ Eccentricity $= e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$

⑥ length of Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 16}{6}$

$= \frac{16}{3}$

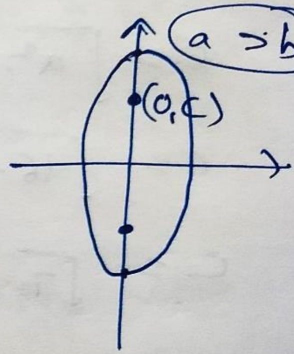
Q.2 $\frac{x^2}{4} + \frac{y^2}{25} = 1$

Larger

Vertical

$a > b$

$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$



$b = 2, \; a = 5$

$c = \sqrt{a^2 - b^2} = \sqrt{25 - 4}$

$c = \sqrt{21}$

① Foci $\Rightarrow (0, \pm c) = (0, \pm\sqrt{21})$

② Vertices $\rightarrow (0, \pm a) = (0, \pm 5)$

③ length of Major Axis $= 2a = 10$

④ ———— minor Axis $= 2b = 4$

⑤ $e = \frac{c}{a} = \frac{\sqrt{21}}{5}$

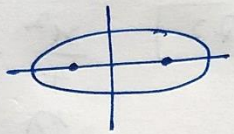
⑥ length of Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{5}$

$= \frac{8}{5}$

Q.3 $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$16 \to a^2$ Larger $\to a = 4$, $9 \to b^2$, $b = 3$

Horizontal



$c = \sqrt{a^2 - b^2}$

$c = \sqrt{16 - 9}$

$c = \sqrt{7}$

① Foci $(\pm c, 0) = (\pm\sqrt{7}, 0)$

② Vertices $(\pm a, 0) = (\pm 4, 0)$

③ length of major axis $= 2a = 8$

④ ——— minor axis $= 2b = 6$

⑤ eccentricity $= e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

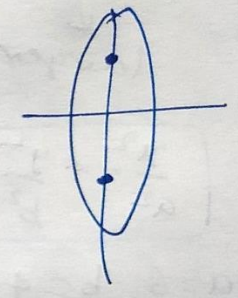
⑥ length of Latus Rectum $= \frac{2b^2}{a}$

$= \frac{2 \times 9}{4} = \frac{9}{2}$

Q.4 $\frac{x^2}{25} + \frac{y^2}{100} = 1$

$25 \to b^2$, $100 \to$ Larger $= a^2$

Vertical



$a = 10$

$b = 5$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{100 - 25}$

$c = \sqrt{3 \times 25} = 5\sqrt{3}$

① Foci $= (0, \pm c) = (0, \pm 5\sqrt{3})$

② vertices $= (0, \pm a) = (0, \pm 10)$

③ length of major axis $= 2a = 20$

④ ——— minor axis $= 2b = 10$

⑤ $e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \sqrt{3}/2$

⑥ length of Latus rectum $= 2\frac{b^2}{a}$

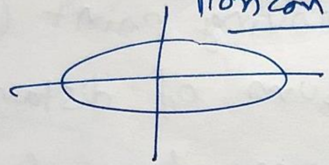
$= \frac{\cancel{2}(25)}{\cancel{10}} = 5$

Q.5 $\frac{x^2}{49} + \frac{y^2}{36} = 1$

$49 \rightarrow$ Larger $= a^2$

$36 \longrightarrow b^2$

Horizontal



$a = 7$

$b = 6$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{49 - 36} = \sqrt{13}$

① Foci $\Rightarrow (\pm c, 0) \longrightarrow (\pm\sqrt{13}, 0)$

② Vertices $\Rightarrow (\pm a, 0) \rightarrow (\pm 7, 0)$

③ length of major Axis $= 2a = 14$

④ ——— minor Axis $= 2b = 12$

⑤ Eccentricity $= e = \frac{c}{a} = \frac{\sqrt{13}}{7}$

⑥ Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(36)}{7}$

$= 72/7$

Q.6 $\frac{x^2}{100} + \frac{y^2}{400} = 1$

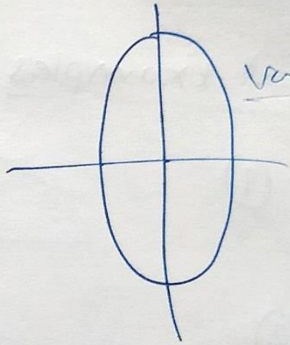
$100 \rightarrow b^2$

$400 \longrightarrow$ Larger $= a^2$

$a = 20$

$b = 10$

Vertical



$c = \sqrt{a^2 - b^2}$

$c = \sqrt{300}$

$c = 10\sqrt{3}$

① Foci $\Rightarrow (0, \pm c) = (0, \pm 10\sqrt{3})$

② Vertices $\Rightarrow (0, \pm a) = (0, \pm 20)$

③ Length of major axis $= 2a = 40$

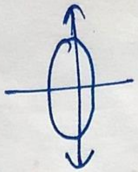
④ ——— minor axis $= 2b = 20$

⑤ $e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \sqrt{3}/2$

⑥ length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2(100)}{20} = 10$

Q.7 $\dfrac{36x^2 + 4y^2 = 144}{144}$



$\Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$

$9 \rightarrow b^2$, $36 \rightarrow$ Larger $= a^2$

$\boxed{b = 3}$ $\boxed{a = 6}$

$c = \sqrt{a^2 - b^2} = \sqrt{36 - 9}$

$c = \sqrt{27} = 3\sqrt{3}$

1) Foci $(0, \pm c) = (0, \pm 3\sqrt{3})$

2) Vertices $(0, \pm a) = (0, \pm 6)$

3) length of major axis $= 2a = 12$

4) ——— minor axis $= 2b = 6$

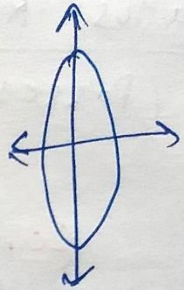
5) Eccentricity $= e = \dfrac{c}{a} = \dfrac{3\sqrt{3}}{6} = \sqrt{3}/2$

6) length of Latus Rectum $= \dfrac{2b^2}{a} = \dfrac{2(9)}{6}$

$= 3$

Q.8 $\cancel{4x^2 + 9y^2 = 36}$

$\dfrac{16x^2 + y^2 = 16}{16}$



$\Rightarrow \dfrac{x^2}{1} + \dfrac{y^2}{16} = 1$

$1 \rightarrow b^2$, $16 \rightarrow$ Larger $= a^2$

$a = 4$

$b^2 = 1$

$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1} = \sqrt{15}$

1) Foci $\rightarrow (0, \pm c) = (0, \pm\sqrt{15})$

2) Vertices $\rightarrow (0, \pm a) = (0, \pm 4)$

3) Length of major axis $= 2a = 8$

4) ——— minor axis $= 2b = 2$

5) $e = \dfrac{c}{a} = \dfrac{\sqrt{15}}{4}$

6) length of Latus Rectum $= \dfrac{2b^2}{a}$

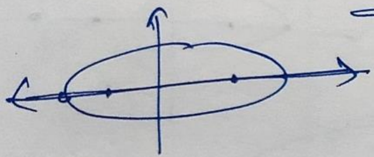
$= \dfrac{2(1)}{4} = \dfrac{1}{2}$

Q.9 $\dfrac{4x^2 + 9y^2 = 36}{36}$

$\Rightarrow \boxed{\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1}$

Larger $= a^2$

$a = 3$
$b = 2$



$c = \sqrt{a^2 - b^2}$

$c = \sqrt{9 - 4}$

$\boxed{c = \sqrt{5}}$

① Foci $(\pm c, 0) \rightarrow (\pm\sqrt{5}, 0)$

② Vertices $(\pm a, 0) = (\pm 3, 0)$

③ length of major axis $= 2a = 6$

④ ———— minor axis $= 2b = 4$

⑤ Eccentricity $= e = \dfrac{c}{a} = \dfrac{\sqrt{5}}{3}$

⑥ Length of Latus Rectum $= \dfrac{2b^2}{a}$

$= \dfrac{2(4)}{3} = 8/3$

Exercise 11.3 Ellipse

Q.10 Vertices $(\pm 5, 0)$ $(\pm a, 0)$

foci $(\pm 4, 0)$ $(\pm c, 0)$

y-coordinate $= 0$

$a = 5$
$c = 4$

Ellipse → Horizontal, $a > b$

$\rightarrow \boxed{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$

?

$a = 5, \ b = 3$

$c = \sqrt{a^2 - b^2}$

$\Rightarrow c^2 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - c^2$

$\Rightarrow b^2 = 25 - 16$

$b^2 = 9$

$b = 3$

Equation

$\boxed{\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1}$

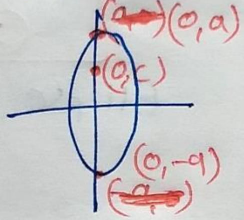
Q.11 Vertices $(0, \pm 13)$ $(0, \pm a)$.

Foci $(0, \pm 5)$ $(0, \pm c)$

x-coordinates $= 0$

$a > b$

Ellipse $\rightarrow$ vertical,



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$a = 13, \ c = 5$

$c = \sqrt{a^2 - b^2}$

$c^2 = a^2 - b^2$

$b^2 = a^2 - c^2$

$b^2 = 169 - 25 = 144$

$b = 12$

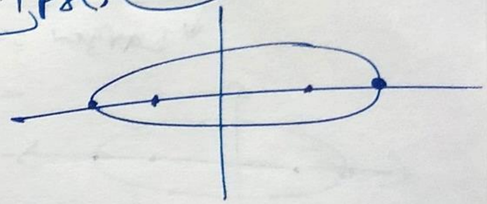
$a = 13$
$b = 12$

$$\boxed{\frac{x^2}{144} + \frac{y^2}{169} = 1}$$

Q.12 Vertices $(\pm 6, 0) \rightarrow (\pm a, 0)$ $\boxed{a = 6}$

foci $(\pm 4, 0) \rightarrow (\pm c, 0)$ $\boxed{c = 4}$

Horizontal Ellipse. $a > b$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$b^2 = a^2 - c^2$

$b^2 = 6^2 - 4^2$

$b^2 = 36 - 16 = 20$

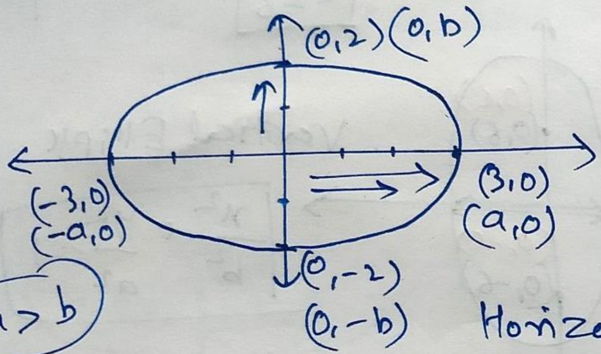
$b = \sqrt{20} = 2\sqrt{5}$

Equation

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

Q.13 Ends of major axis $(\pm 3, 0)$

Ends of minor axis $(0, \pm 2)$



$a > b$

Horizontal Ellipse

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$a = 3$

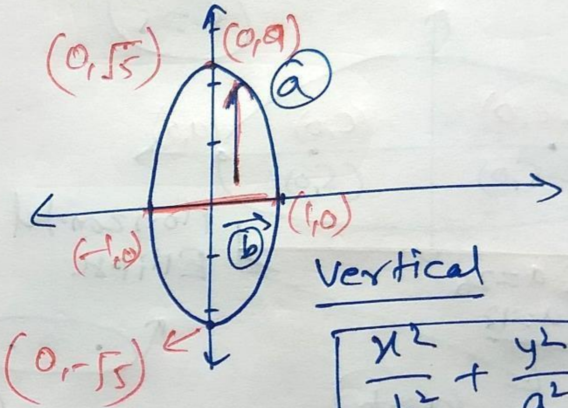
$b = 2$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{4} = 1}$$

Q.14 Ends of major axis $(0, \pm\sqrt{5})$

Ends of minor axis $(\pm 1, 0)$

$\sqrt{5} \approx 2.236$



Vertical

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

$a = \sqrt{5}$ $b = 1$

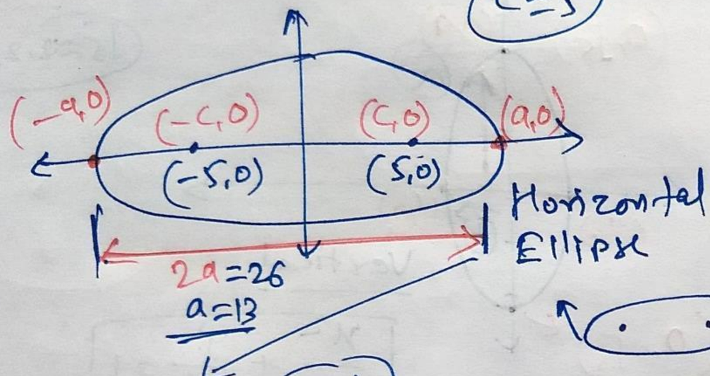
$$\Rightarrow \frac{x^2}{1^2} + \frac{y^2}{(\sqrt{5})^2} = 1$$

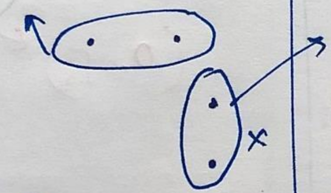
$$\Rightarrow \boxed{\frac{x^2}{1} + \frac{y^2}{5} = 1}$$

Q.15 length of major Axis $= 26 = 2a$

foci $= (\pm 5, 0) \equiv (\pm c, 0)$

$c = 5$





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a = 13$

$c = 5$

$c = \sqrt{a^2 - b^2}$

$b^2 = a^2 - c^2$

$b^2 = 169 - 25$

$b^2 = 144$

$$\Rightarrow \boxed{\frac{x^2}{169} + \frac{y^2}{144} = 1}$$

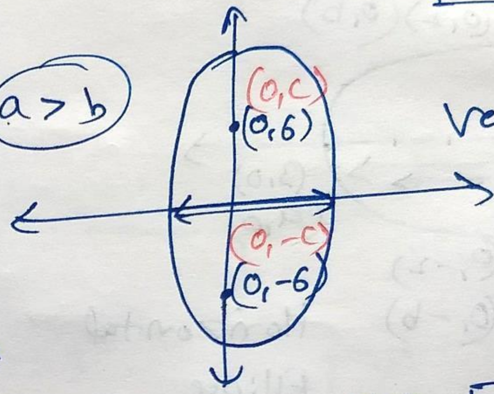
Q.16 length of minor axis $= 16 = 2b$

$b = 8$

foci $= (0, \pm 6) \equiv (0, \pm c)$

$\boxed{c = 6}$

$a > b$



Vertical Ellipse

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

$c = \sqrt{a^2 - b^2}$

$\Rightarrow c^2 = a^2 - b^2$

$\Rightarrow a^2 = c^2 + b^2$

$\Rightarrow a^2 = 6^2 + 8^2$

$\Rightarrow a^2 = 36 + 64 = 100$

$a = 10$

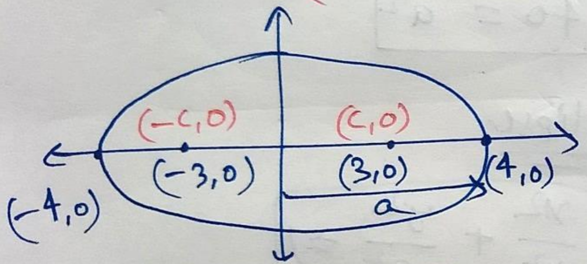
Equation of Ellipse

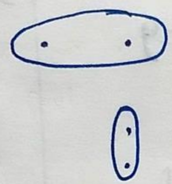
$$\boxed{\frac{x^2}{64} + \frac{y^2}{100} = 1}$$

Ellipse

Q.17 Foci $(\pm 3, 0)$, $a = 4$

$(\pm c, 0)$





Horizontal Ellipse $\quad a > b$

$\boxed{c = 3}$ $\quad (a = 4)$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$$c = \sqrt{a^2 - b^2}$$

$$\Rightarrow b^2 = a^2 - c^2$$

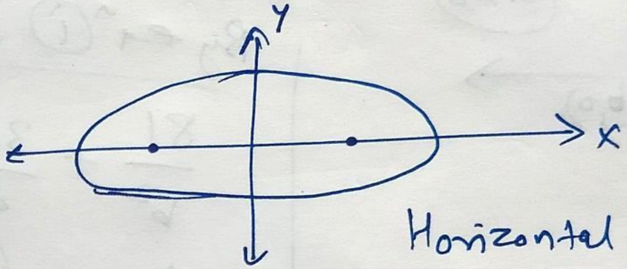
$$\Rightarrow b^2 = 16 - 9$$

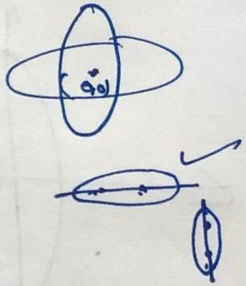
$$\boxed{b^2 = 7}$$

$$\boxed{\frac{x^2}{16} + \frac{y^2}{7} = 1}$$

Q.18 $b = 3$, $c = 4$, Centre $(0, 0)$

Foci on x-axis





Horizontal

$a > b$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$b = 3$

$c = 4$

$$c = \sqrt{a^2 - b^2}$$

$$\Rightarrow c^2 = a^2 - b^2$$

$$\Rightarrow c^2 + b^2 = a^2$$

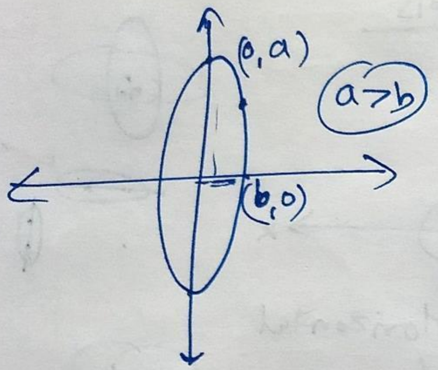
$$\Rightarrow 16 + 9 = a^2$$

$$\Rightarrow \boxed{a^2 = 25}$$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

$\boxed{Q.19}$ Centre $(0,0)$, Major Axis = y-axis

Passes through $(3,2)$ & $(1,6)$



$\boxed{a > b}$

Vertical Ellipse

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

(I) $(3,2)$

$$\Rightarrow \boxed{\frac{9}{b^2} + \frac{4}{a^2} = 1} \times 9 \quad (1)$$

(II) $(1,6)$

$$\boxed{\frac{1}{b^2} + \frac{36}{a^2} = 1} \quad (2)$$

By eqn (1) & (2):→

$$\frac{81}{b^2} + \cancel{\frac{36}{a^2}} = 9$$

$$\frac{1}{b^2} + \cancel{\frac{36}{a^2}} = 1$$

$$- \qquad - \qquad -$$

$$\frac{\cancel{80}^{10}}{b^2} = \cancel{8}$$

$$\Rightarrow \boxed{10 = b^2}$$

By eqn (2):

$$\frac{1}{10} + \frac{36}{a^2} = 1$$

$$\Rightarrow \frac{36}{a^2} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\Rightarrow \frac{\cancel{36}^{4}}{a^2} = \frac{\cancel{9}}{10}$$

$$\Rightarrow \boxed{40 = a^2}$$

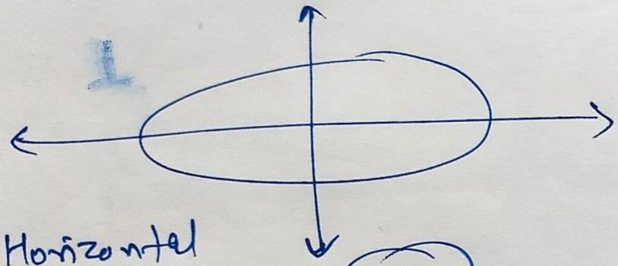
Ellipse

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

$$\Rightarrow \boxed{\frac{x^2}{10} + \frac{y^2}{40} = 1}$$

Q.20 Major Axis → x-axis

Passes through (4,3) & (6,2)



Horizontal

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$ $a>b$

Large Small

I (4,3) $\left(\frac{16}{a^2}+\frac{9}{b^2}=1\right)$ — ① ×4

II (6,2) $\left(\frac{36}{a^2}+\frac{4}{b^2}=1\right)$ — ② ×9

By eqⁿ ① & ②

$$\frac{64}{a^2}+\cancel{\frac{36}{b^2}}=4$$

$$\frac{324}{a^2}+\cancel{\frac{36}{b^2}}=9$$

$$- \qquad - \qquad -$$

$$\Rightarrow \frac{64-324}{a^2}=4-9$$

$$\Rightarrow \frac{\cancel{-260}^{52}}{a^2}=\cancel{-}\cancel{5}$$

$$\Rightarrow \boxed{a^2=52}$$

By eqⁿ ②

$$\frac{\cancel{36}^{9}}{\cancel{52}_{13}}+\frac{4}{b^2}=1$$

$$\Rightarrow \frac{9}{13}+\frac{4}{b^2}=1$$

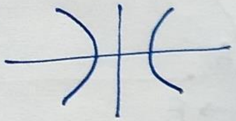
$$\frac{4}{b^2}=1-\frac{9}{13}$$

$$\Rightarrow \frac{4}{b^2}=\frac{13-9}{13}=\frac{4}{13}$$

$$\Rightarrow \boxed{b^2=13}$$

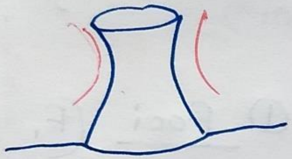
$$\boxed{\frac{x^2}{52}+\frac{y^2}{13}=1}$$

Hyperbola before Exercise 10.4



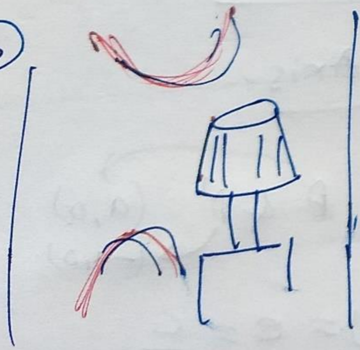
Real life Examples.

1) Nuclear Reactor



2) Chair

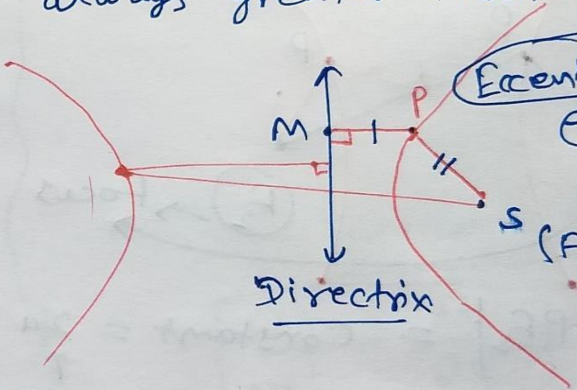
3)



4)

5)

Definition - I Hyperbola is the locus of a moving point 'P' such that the ratio of distance b/w 'P' & a fixed point 'S' (Focus) & a fixed line (Directrix) is constant and always greater than 1.

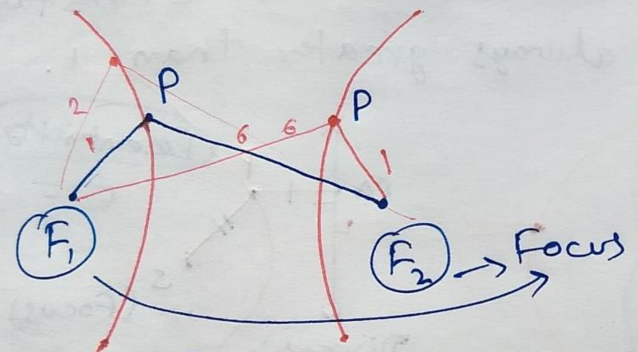


Eccentricity

$$e = \frac{SP}{PM} = \text{Constant} > 1$$

SP > PM

Definition - ② Hyperbola is the locus of a moving point(P) from where difference of distances to the two fixed points (foci-F_1 & F_2) is always constant.

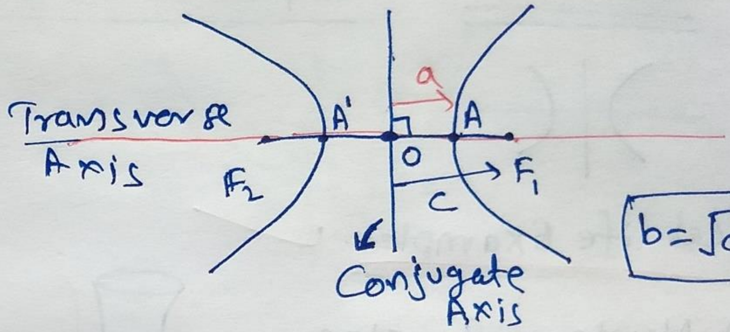


$$|PF_1 - PF_2| = \text{Constant} = 2a$$

(for Derivation)

$$|1-6| = |6-1|$$

Terms Related to HYPERBOLA



$$\boxed{b = \sqrt{c^2 - a^2}}$$

① Foci (F_1 & F_2) $(c, 0), (-c, 0)$

② Centre. F_1F_2 का mid point

③ Transverse Axis.

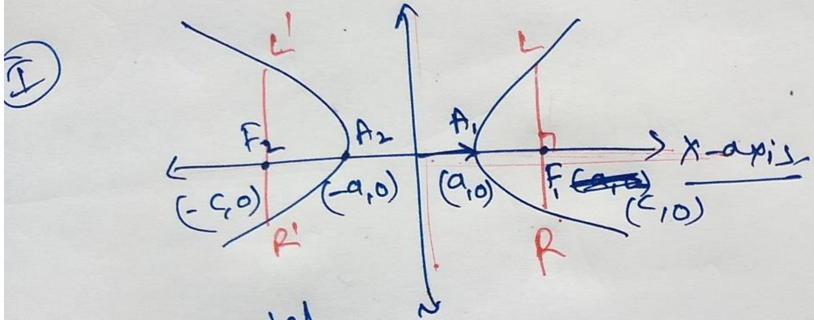
④ Conjugate Axis.

⑤ Vertices A & A' $(a, 0)$, $(-a, 0)$

⑥ eccentricity $= e = \dfrac{c}{a}$

Equations of Hyperbola :
(standard)

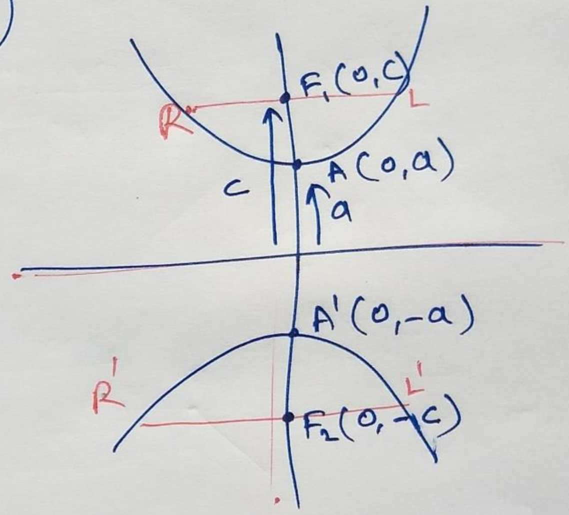
I



Horizontal Hyp.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

II



Vertical Hyp.

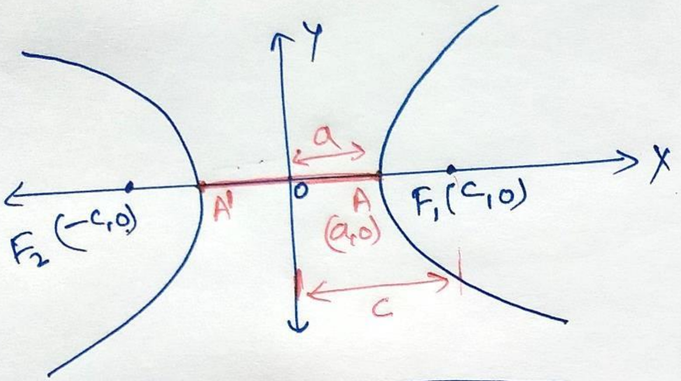
$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$

Note : If $a = b$ $\quad \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

$$\Rightarrow \boxed{x^2 - y^2 = a^2}$$

Equilateral Hyperbola (Rectangular Hyperbola)

Horizontal Hyperbola



EQUATION $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

Centre (0,0)

Transverse Axis x-axis → AA' = 2a

Conjugate Axis y-axis → 2b

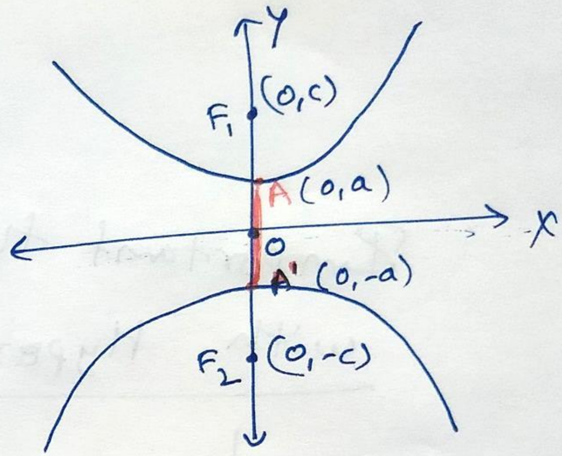
Vertices A & A' (±a,0)

Foci F_1, F_2 (±c,0)

length of latus Rectum = $\frac{2b^2}{a}$

$e = \frac{c}{a}$ $c = \sqrt{a^2+b^2}$ → $b = \sqrt{}$

Vertical Hyperbola



EQUATION $\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$

Centre (0,0)

Transverse Axis → y-axis → 2a = AA'

Conjugate Axis → x-axis → 2b

Vertices A & A' (0,±a)

Foci F_1 & F_2 (0,±c)

length of latus Rectum = $\frac{2b^2}{a}$

$e = \frac{c}{a}$ $c = \sqrt{a^2+b^2}$ $b = \sqrt{}$

Important things to deal with Hyperbola Questions

- Diagram
- any two out of <u>a, b, c</u>

(Hyperbola)

Q.1 $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

Horizontal Hyp. $\boxed{\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1}$

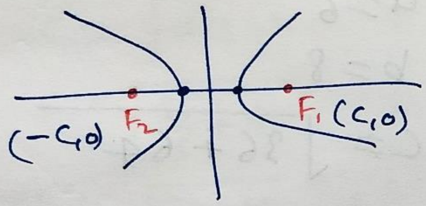
$a = 4, \quad b = 3$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{16 + 9}$

$c = \sqrt{25}$

$c = 5$



coordinates of Foci (F_1 & F_2) $= (\pm c, 0) \to (\pm 5, 0)$

Vertices $(\pm a, 0) \equiv (\pm 4, 0)$

Eccentricity $= e = \dfrac{c}{a} = \dfrac{5}{4}$

length of Latus Rectum $= \dfrac{2b^2}{a} = \dfrac{2(9)}{4}$

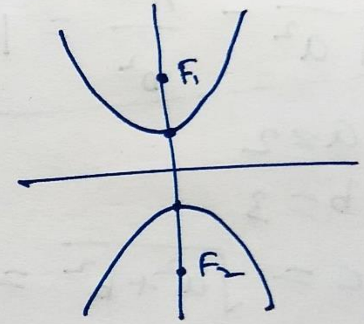
$= \dfrac{9}{2}$ units

Q.2 $\dfrac{y^2}{9} - \dfrac{x^2}{27} = 1$

$+ \to$ Vertical Hyp. $\boxed{\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1}$

$a = 3$

$b = 3\sqrt{3}$



$c = \sqrt{a^2 + b^2}$

$c = \sqrt{9 + 27}$

$c = \sqrt{36}$

$\boxed{c = 6}$

Foci $(0, \pm c) \to (0, \pm 6)$

Vertices $(0, \pm a) \to (0, \pm 3)$

Eccentricity $= e = \dfrac{c}{a} = \dfrac{6}{3} = 2$

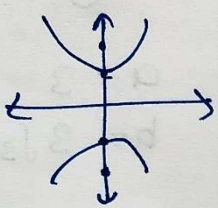
length of Latus Rectum $= \dfrac{2b^2}{a} = \dfrac{2(\cancel{27})^{9}}{\cancel{3}}$

$= 18$

Q.3 $\quad \dfrac{9y^2 - 4x^2 = 36}{36}$

$\Rightarrow \boxed{\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1}$ Vertical Hyp.

$\boxed{\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1}$



$a = 2$

$b = 3$

$c = \sqrt{a^2+b^2} = \sqrt{4+9} = \sqrt{13}$

Foci $(0, \pm c) \equiv (0, \pm\sqrt{13})$

Vertices $(0, \pm a) \equiv (0, \pm 2)$

Eccentricity $= e = \dfrac{c}{a} = \dfrac{\sqrt{13}}{2}$

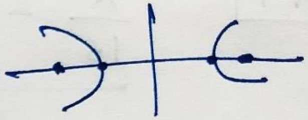
length of Latus Rectum $= \dfrac{2b^2}{a} = \dfrac{2(9)}{2}$

$= 9$

Q.4 $\quad \dfrac{16x^2 - 9y^2 = 576}{576}$

$\Rightarrow \dfrac{x^2}{36} - \dfrac{y^2}{64} = 1$

Horizontal



$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$a = 6$

$b = 8$

$c = \sqrt{36+64} = \sqrt{100} = 10$

Foci $\rightarrow (\pm c, 0) \equiv (\pm 10, 0)$

Vertices $\rightarrow (\pm a, 0) \equiv (\pm 6, 0)$

Eccentricity $= e = \dfrac{c}{a} = \dfrac{10}{6} = \dfrac{5}{3}$

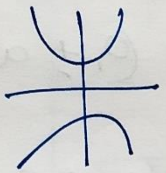
length of Latus Rectum $= \dfrac{2b^2}{a} = \dfrac{\cancel{2}(64)}{\cancel{6}\,3}$

$= \dfrac{64}{3}$

Q.5 $\dfrac{5y^2 - 9x^2 = 36}{36}$

$\Rightarrow \dfrac{5y^2}{36} - \dfrac{x^2}{4} = 1$ Vertical

$\Rightarrow \boxed{\dfrac{y^2}{\left(\frac{36}{5}\right)} - \dfrac{x^2}{(4)} = 1}$



$\boxed{\dfrac{y^2}{(a^2)} - \dfrac{x^2}{b^2} = 1}$

$a = \dfrac{6}{\sqrt{5}}$, $b = 2$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{\dfrac{36}{5} + 4}$

$c = \sqrt{\dfrac{36+20}{5}} = \sqrt{\dfrac{56}{5}}$

$2\sqrt{14}/\sqrt{5}$

Foci $\to (\pm c, 0) \to \left(\pm\sqrt{\dfrac{56}{5}}, 0\right)$

Vertices $\to (\pm a, 0) \to \left(\pm\dfrac{6}{\sqrt{5}}, 0\right)$

Eccentricity $= e = \dfrac{c}{a} = \dfrac{\sqrt{56}}{\sqrt{5}} \Big/ \dfrac{6}{\sqrt{5}} = \dfrac{\sqrt{56}}{6}$

Length of Latus Rectum $= \dfrac{2b^2}{a}$

$= \dfrac{2(4)}{\left(\frac{6}{\sqrt{5}}\right)} = \dfrac{4\sqrt{5}}{3}$

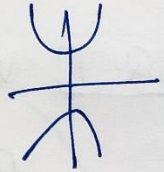
$(0, \pm a)$

$(0, \pm c)$ Correction

Q.6 $\dfrac{49y^2 - 16x^2 = 784}{784}$

$\Rightarrow \boxed{\dfrac{y^2}{16} - \dfrac{x^2}{49} = 1}$ Vertical Hyp.

$\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$



$a = 4$

$b = 7$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{16 + 49} = \sqrt{65}$

Foci $(0, \pm c) \equiv (0, \pm\sqrt{65})$

Vertices $(0, \pm a) \equiv (0, \pm 4)$

Eccentricity $= e = \dfrac{c}{a} = \dfrac{\sqrt{65}}{4}$

Length of Latus Rectum $= \dfrac{2b^2}{a} = \dfrac{\cancel{2}(49)}{\cancel{4}_2}$

$= \dfrac{49}{2}$

Hyperbola

Q.7 Vertices $(\pm 2, 0)$

Foci $(\pm 3, 0)$

$y = 0$

Horizontal Hyp.

$(\pm a, 0)$ $a = 2$

$(\pm c, 0)$ $\boxed{c = 3}$

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{3^2 - 2^2}$$

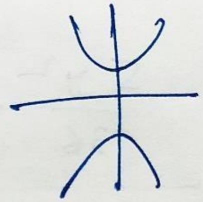
$$b = \sqrt{9 - 4} = \sqrt{5}$$

Equation

$$\boxed{\frac{x^2}{4} - \frac{y^2}{5} = 1}$$

Q.8 Vertices $(0, \pm 5)$

Vertical Hyp.



Foci $(0, \pm 8)$

$(0, \pm a)$ $\boxed{a = 5}$

$(0, \pm c)$ $\boxed{c = 8}$

$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{64 - 25}$$

$$b = \sqrt{39}$$

$$b^2 = 39$$

Equation,

$$\boxed{\frac{y^2}{25} - \frac{x^2}{39} = 1}$$

Q.9

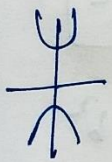
Vertices $(0, \pm 3) \equiv (0, \pm a)$

Foci $(0, \pm 5) \equiv (0, \pm c)$

Vertical Hyp.

$\boxed{a = 3}$

$\boxed{c = 5}$



$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{25 - 9}$$

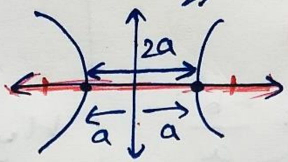
$$b = \sqrt{16} = 4$$

Eq^n, $$\boxed{\frac{y^2}{9} - \frac{x^2}{16} = 1}$$

Q.10 Foci $(\pm 5, 0)$ $(\pm c, 0)$

length (transverse Axis) $= 8 = 2a$

Hyp.



$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

$\boxed{c = 5}$

$\boxed{a = 4}$

$$b = \sqrt{c^2 - a^2} = \sqrt{25 - 16}$$

$$\boxed{b = \sqrt{9} = 3}$$

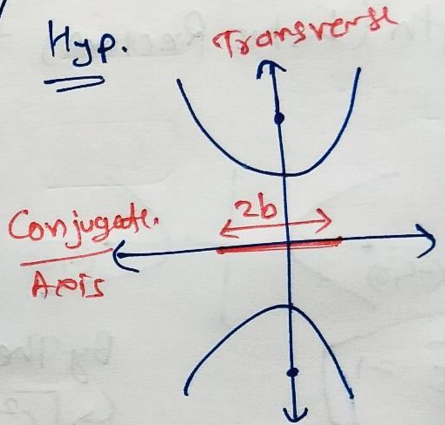
Eq^n. $$\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$$

Q.11

Foci $(0, \pm 13) \Leftrightarrow (0, \pm c)$

Length (Conjugate Axis) $= 24 = 2b$

Hyp.



$c = \sqrt{a^2 + b^2}$

$\boxed{c = 13}$

$\boxed{b = 12}$

Eq^n. $$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$

$$c^2 = a^2 + b^2$$

$$\Rightarrow c^2 - b^2 = a^2$$

$$\Rightarrow a^2 = 169 - 144$$

$$\Rightarrow \boxed{a^2 = 25}$$

Eq^n. $$\boxed{\frac{y^2}{25} - \frac{x^2}{144} = 1}$$

Hyperbola

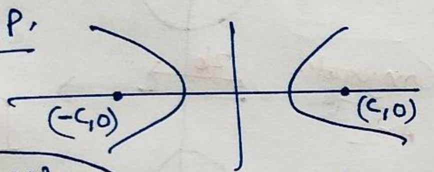
Q.12 Foci $(\pm 3\sqrt{5}, 0) \equiv (\pm c, 0)$

length (Latus Rectum) $= 8 = \dfrac{2b^2}{a}$

$4a = b^2$

$\boxed{c = 3\sqrt{5}}$

Hyp.



$\boxed{\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1}$

By Theory

$\hookrightarrow \boxed{c^2 = a^2 + b^2}$

$\Rightarrow (3\sqrt{5})^2 = a^2 + 4a$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow \underbrace{a^2 + 9a} - \underbrace{5a - 45} = 0$

$\Rightarrow a(a+9) - 5(a+9) = 0$

$\Rightarrow (a-5)(a+9) = 0$

$\boxed{a = 5}$ $\cancel{a = -9}$

$a = 5$ ✓

$b^2 = 4a$

$b^2 = 20$ ✓

Eqn.

$\boxed{\dfrac{x^2}{25} - \dfrac{y^2}{20} = 1}$ ✓

Q.13 Foci $(\pm 4, 0) \equiv (\pm c, 0)$ $\boxed{c = 4}$

length $\left(\begin{array}{l}\text{Latus}\\ \text{Rectum}\end{array}\right) = 12 = \dfrac{2b^2}{a}$

$\boxed{6a = b^2}$

Hyp.

$\boxed{\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1}$

$c^2 = a^2 + b^2$

$\Rightarrow 16 = a^2 + 6a$

$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow \underbrace{a^2 + 8a} - \underbrace{2a - 16} = 0$

$\Rightarrow (a+8)(a-2) = 0$

$\cancel{a = -8}$ $a = 2$ ✓

Eqn.

$\boxed{\dfrac{x^2}{4} - \dfrac{y^2}{12} = 1}$

$a = 2$ ✓, $b^2 = 6a$

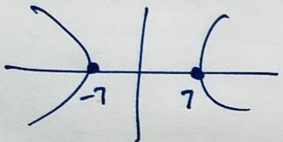
$b^2 = 12$ ✓

Q.14 Vertices $(\pm 7, 0)$ $(\pm a, 0)$

$\boxed{a = 7}$

$e = \frac{4}{3} = \frac{c}{a}$

Hyp.



$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

$c = \frac{4a}{3}$

$c = \frac{4 \times 7}{3} = \frac{28}{3}$

$b = \sqrt{c^2 - a^2}$

$b^2 = \left(\frac{28}{3}\right)^2 - 49$

$b^2 = \frac{784}{9} - 49$

$b^2 = \frac{784 - 441}{9}$

$b^2 = \frac{343}{9}$

Equation

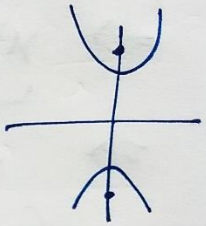
$\frac{x^2}{49} - \frac{y^2}{\left(\frac{343}{9}\right)} = 1$

Q.15 Foci $(0, \pm\sqrt{10}) \leftrightarrow (0, \pm c)$ $\boxed{c = \sqrt{10}}$

Passing through $(2, 3)$

Vertical Hyp.

$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$



put $(2,3) \rightarrow$

$\Rightarrow \frac{9}{(a^2)} - \frac{4}{b^2} = 1$ —(2)

$c^2 = a^2 + b^2$

$\Rightarrow 10 = a^2 + b^2$

$\Rightarrow \boxed{a^2 = 10 - b^2}$ —(1)

Substitute

$\Rightarrow \frac{9}{10 - b^2} - \frac{4}{b^2} = 1$

$b^2 = x$

$\Rightarrow \frac{9}{10 - x} - \frac{4}{x} = 1$

$\Rightarrow \frac{9x - (40 - 4x)}{x(10 - x)} = 1$

$\Rightarrow 13x - 40 = 10x - x^2$

$\Rightarrow \boxed{x^2 + 3x - 40 = 0}$

$\Rightarrow \quad x^2 + 3x - 40 = 0$

$x = b^2$

$\Rightarrow \quad \underbrace{x^2 + 8x} - 5x - 40 = 0$

$\Rightarrow \quad x(x+8) - 5(x+8) = 0$

$(x-5)(x+8) = 0$

$b^2 = x = 5$ ✓ $\qquad b^2 = x = -8$ ✗

$\boxed{b^2 = 5}$

By eqn ① $\quad a^2 = 10 - (5)$

$\boxed{a^2 = 5}$

Eqn. of Hyp.

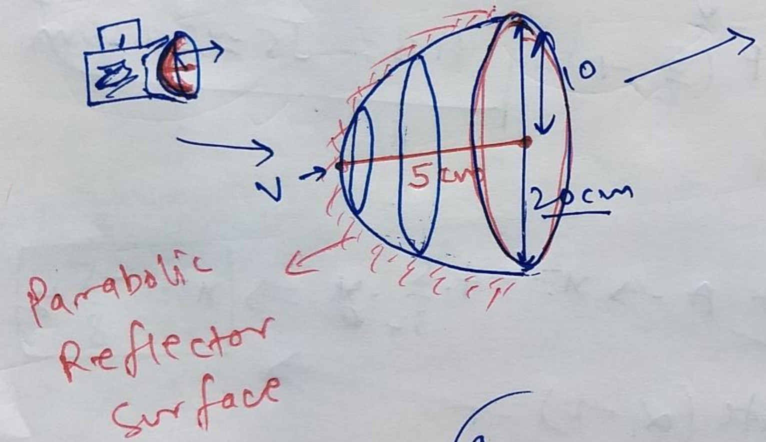
$$\boxed{\frac{y^2}{(5)} - \frac{x^2}{(5)} = 1}$$

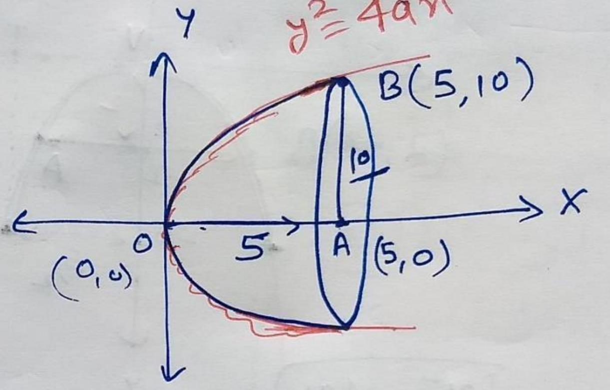
✓

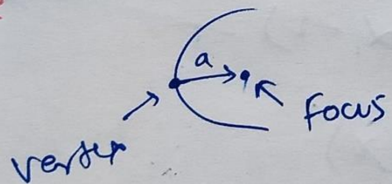
Q.1 Parabolic Reflector (Torch)

Diameter = 20 cm

Deep 5cm







$\because$ B(5,10) lies on Parabola $y^2 = 4ax$

$\Rightarrow 100 = 4 \cdot a \cdot 5$

$\Rightarrow \frac{100}{20} = a$

$\Rightarrow \boxed{a = 5}$

$\boxed{y^2 = 20x}$

vertex (0,0)

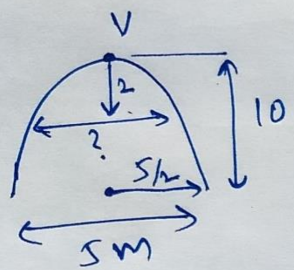
Focus (5,0) → A
(a,0) on

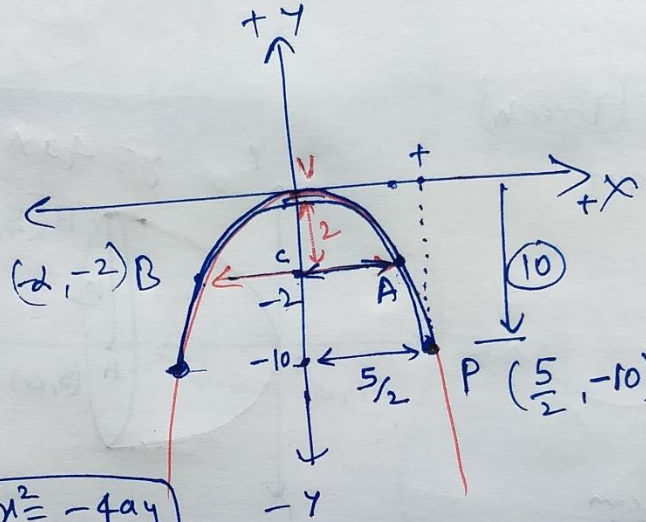
Q.2 Parabolic Arch

Axis Vertical

10 m high, 5 m wide at Base

How wide is it 2 m from vertex ?





$A(\alpha, -2) \equiv A\left(\frac{\sqrt{5}}{2}, -2\right)$

$CA = \frac{\sqrt{5}}{2}$

$AB = 2 \times CA$

$AB = 2 \times \sqrt{5}/2$

$AB = \sqrt{5} \approx 2.236\, m$

$\boxed{x^2 = -4ay}$

$\because P\left(\frac{5}{2}, -10\right)$ lies on

$\Rightarrow \frac{\cancel{25}\, 5}{4} = \cancel{+}\, 4a(\cancel{+}\cancel{10}\,2)$

$\Rightarrow \frac{5}{32} = a$

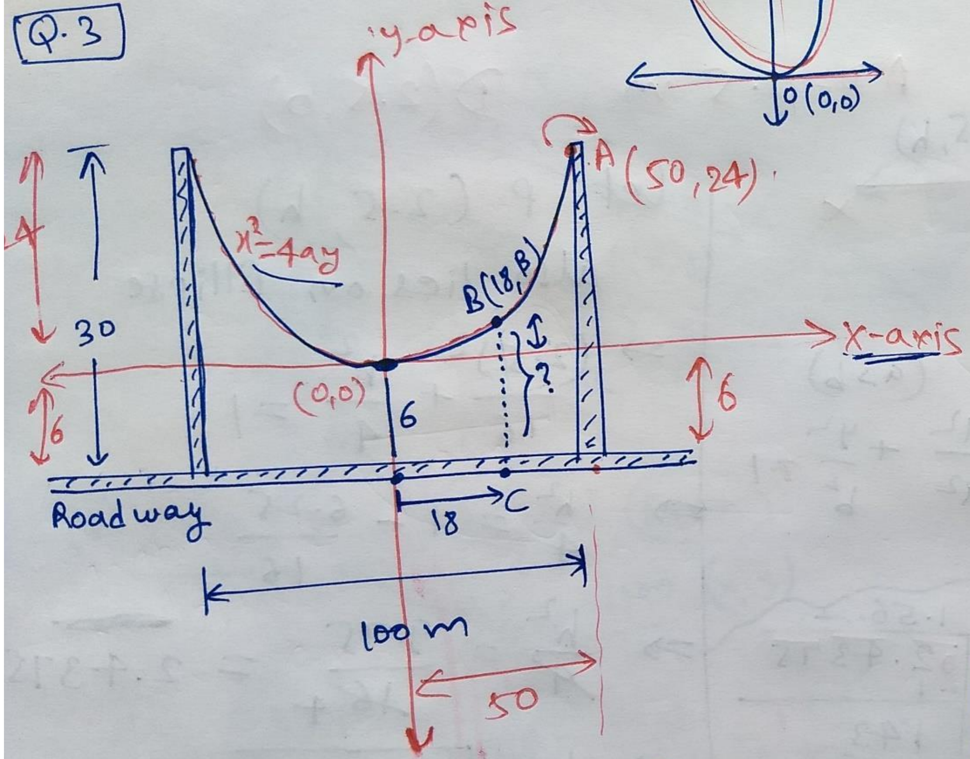
For Point A $\rightarrow x^2 = -4 \cdot \frac{5}{32} \cdot y \Rightarrow \boxed{x^2 = \frac{-5}{8} y}$

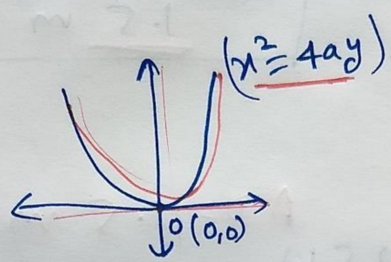
Coordinate $(\alpha, -2)$

$\alpha^2 = \cancel{+}\frac{5}{\cancel{8}\,4}(\cancel{+}\cancel{2})$

$\alpha^2 = \frac{5}{4} \Rightarrow \left(\alpha = \pm \frac{\sqrt{5}}{2}\right)$

Q.3





A(50, 24) lies on $x^2 = 4ay$

$$\Rightarrow (50)^2 = 4a(24)$$

$$\Rightarrow \frac{\cancel{2500}^{\,625}}{\cancel{4} \times 24} = a$$

$$\Rightarrow \boxed{a = \frac{625}{24}}$$

Parabola $x^2 = 4ay = \cancel{4} \times \frac{625}{\cancel{24}_{\,6}} \cdot y$

$$\Rightarrow \boxed{x^2 = \frac{625}{6} y}$$

B? → let B be $(18, \beta)$ satisfy $\left(x^2 = \frac{625}{6} y\right)$

$$\Rightarrow 324 = \frac{625}{6} \cdot \beta$$

$$\Rightarrow \beta = \frac{324 \times 6}{625} \approx 3.1104 \text{ m}$$

$B(18, 3.1104)$

BC = y coordinate of B + '6'

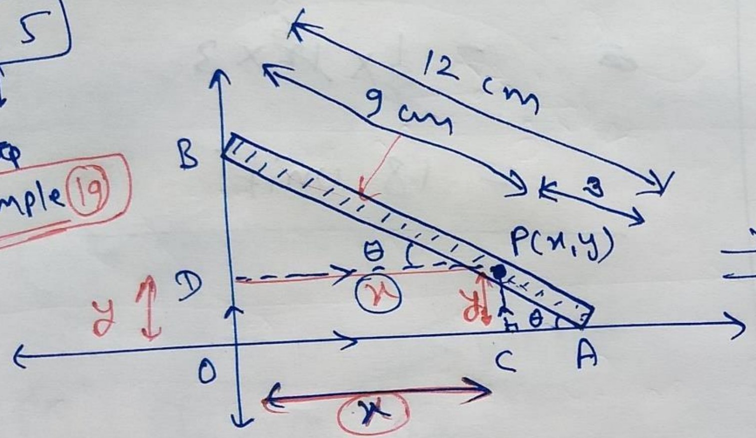
$= 3.1104 + 6$

BC = 9.1104 m

Q. 5

~~Examp~~

Example 19



Let P(x, y)

Locus

Path

Equation (x, y)

In ΔACP $\sin\theta = \frac{Perp.}{Hyp.} = \frac{CP}{AP} = \frac{y}{3}$

$\Rightarrow$ $\cancel{\boxed{y = 3\sin\theta}}$

In ΔPDB.

$\cos\theta = \frac{Base}{Hyp.} = \frac{PD}{PB} = \frac{x}{9}$

Arbitrary Constanst

assume

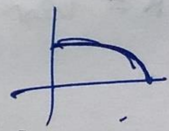
$\sin\theta = \frac{y}{3}$, $\cos\theta = \frac{x}{9}$

Identity $\sin^2\theta + \cos^2\theta = 1$

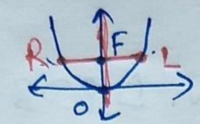
$\Rightarrow \left(\frac{y}{3}\right)^2 + \left(\frac{x}{9}\right)^2 = 1$

$\Rightarrow \boxed{\frac{x^2}{81} + \frac{y^2}{9} = 1}$

P(x, y) Ellipse



Q.6 $\boxed{x^2 = 12y}$

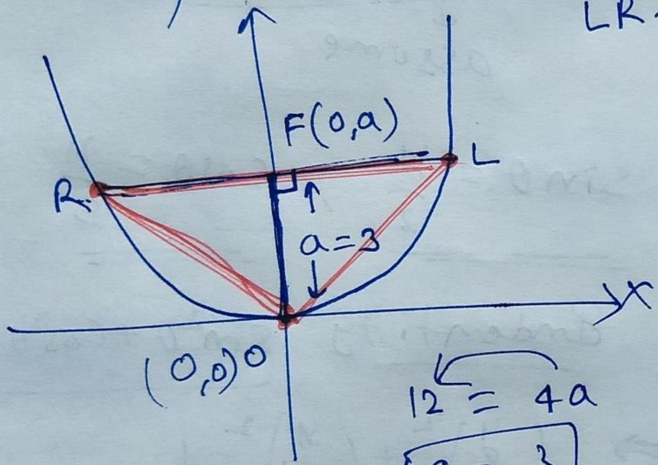


$\boxed{x^2 = 4ay}$

Triangle area → 3 point — Latus Rectum, Vertex

Δ.

LR → Latus Rectum



$12 = 4a$

$\boxed{a = 3}$

Focus $(0,a) \equiv (0,3)$

LR = length of latus Rectum $= 4a = 12$

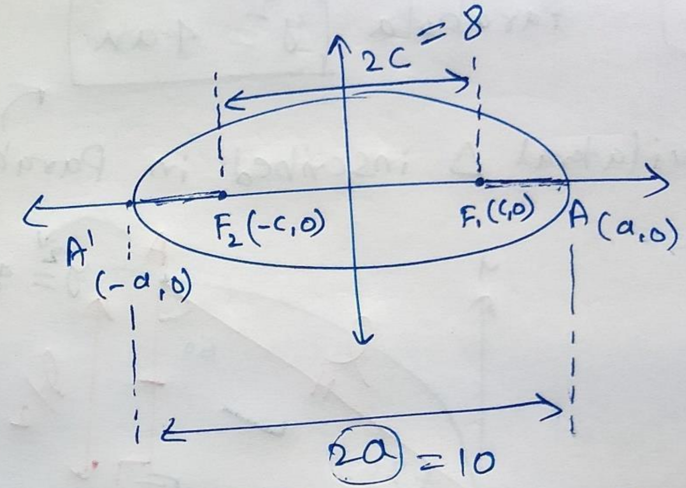
$\boxed{LR = 12}$

area of $\Delta OLR = \frac{1}{2} \times \underline{Base} \times \underline{Height}$

$= \frac{1}{2} \times LR \times OF$

$= \frac{1}{\cancel{2}} \times \cancel{12}^{6} \times 3$

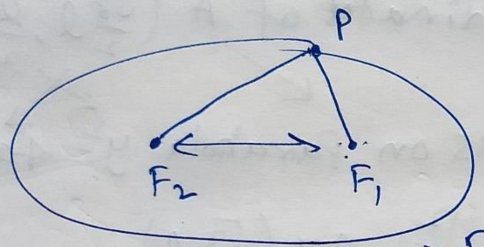
$= 18 \text{ unit}^2$



$\boxed{2a} = 10$

$\boxed{a = 5}$ $\boxed{c = 4}$

$\boxed{Q.7}$

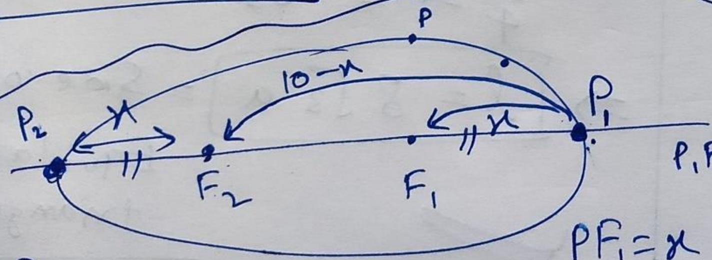


→ Ellipse.

$\boxed{PF_1 + PF_2 = 10} = 2a$

$\boxed{F_1F_2 = 8}$ ✓

$\boxed{PF_1 + PF_2 = 2a}$ ☆



$P_1F_1 = P_2F_2$

$PF_1 = x$

$P_2P_1 = \cancel{x} + 10 - \cancel{x} = 10$ ∴ $PF_2 = 10 - x$

$\because c = \sqrt{a^2 - b^2}$

$\Rightarrow c^2 = a^2 - b^2$

$\Rightarrow \quad b^2 = a^2 - c^2$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b^2 = 9$

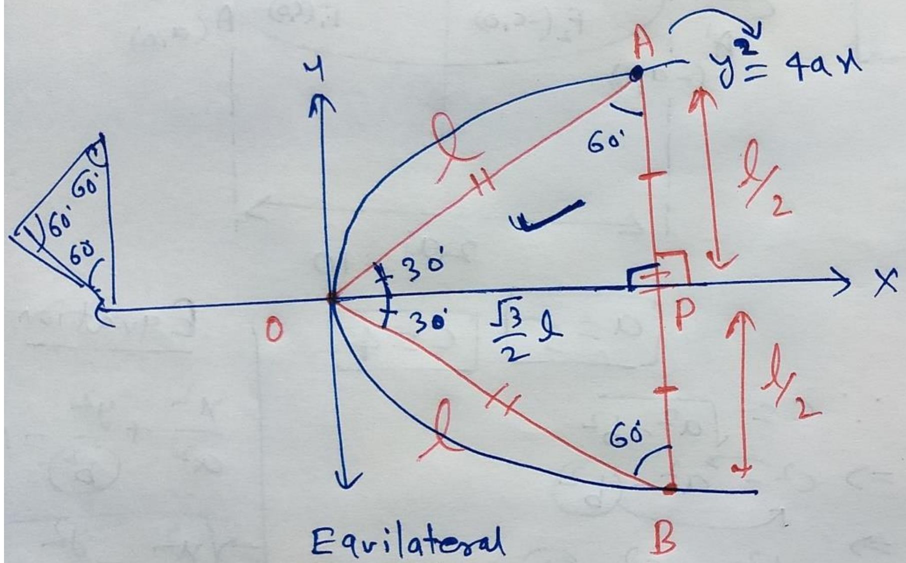
Equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

Q.8 Parabola $\boxed{y^2 = 4ax}$

Equilateral Δ inscribed in Parabola



Side of $\overset{\text{Equilateral}}{\Delta OAB} = l$

In ΔOPA: $\cos 30° = \frac{\text{Base}}{\text{Hyp.}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OP}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OP}{l}$

$\Rightarrow OP = \frac{\sqrt{3}}{2} l$

Coordinates of A $\left(\frac{\sqrt{3}}{2} l, \frac{l}{2}\right)$

lies on Parabola $y^2 = 4ax$

$\Rightarrow \left(\frac{l}{2}\right)^2 = 4a\left(\frac{\sqrt{3}}{2} l\right)$

$\Rightarrow \frac{\cancel{l^2}\, l}{4} = \cancel{4}^{2} . a . \frac{\sqrt{3}}{\cancel{2}} . \cancel{l}$

$\Rightarrow \frac{l}{4} = 2a\sqrt{3}$

$\Rightarrow \boxed{l = 8\sqrt{3}\,a}$ = Side of the Equilateral triangle